

# **NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE**

*(Accredited by NAAC, Approved by AICTE New Delhi, Affiliated to APJKTU)*

**Pampady, Thiruvilwamala(PO), Thrissur(DT), Kerala 680 588**

**DEPARTMENT OF MECHANICAL ENGINEERING**



## **COURSE MATERIALS**



## **EE 311 ELECTRICAL DRIVES AND CONTROL FOR AUTOMATION**

### **VISION OF THE INSTITUTION**

To mould our youngsters into Millennium Leaders not only in Technological and Scientific Fields but also to nurture and strengthen the innate goodness and human nature in them, to equip them to face the future challenges in technological break troughs and information explosions and deliver the bounties of frontier knowledge for the benefit of humankind in general and the down-trodden and underprivileged in particular as envisaged by our great Prime Minister Pandit Jawaharlal Nehru

### **MISSION OF THE INSTITUTION**

To build a strong Centre of Excellence in Learning and Research in Engineering and Frontier Technology, to facilitate students to learn and imbibe discipline, culture and spirituality, besides encouraging them to assimilate the latest technological knowhow and to render a helping hand to the under privileged, thereby acquiring happiness and imparting the same to others without any reservation whatsoever and to facilitate the College to emerge into a magnificent and mighty launching pad to turn out technological giants, dedicated research scientists and intellectual leaders of the society who could prepare the country for a quantum jump in all fields of Science and Technology

## **Vision of the Department of Mechanical Engineering**

Producing internationally competitive Mechanical Engineers with social responsibilities and sustainable employability through viable strategies as well as competent exposure oriented quality education.

## **Mission of the Department of Mechanical Engineering**

**M1:** Imparting high impact education by providing conducive teaching learning environment.

**M2:** Fostering effective modes of continuous learning process with moral and ethical values.

**M3:** Enhancing leadership qualities with social commitment, professional attitude, unity, team spirit and communication skill.

**M4:** Introducing present scenario in research and development through collaborative efforts blended with industry and institution.

## **Program Educational Objectives of the Department of Mechanical Engineering**

**PEO1:** Graduates shall have strong practical and technical exposures in the field of Mechanical Engineering and will contribute to the society through Innovation and Enterprise.

**PEO2:** Graduates will have the demonstrated ability to analyze, formulate and solve design engineering/thermal engineering/materials and manufacturing/design issues and real life problems.

**PEO3:** Graduates will be capable of pursuing Mechanical Engineering profession with good communication skills, leadership qualities, team spirit and professional ethics.

**PEO4:** Graduates will sustain an appetite for continuous learning by pursuing higher education and research in the allied areas of technology

Course code.	Course Name	L-T-P - Credits	Year of Introduction
EE311	ELECTRICAL DRIVES & CONTROL FOR AUTOMATION	3-0-0-3	2016
<b>Prerequisite : Nil</b>			
<b>Course Objectives</b>			
<ol style="list-style-type: none"> <li>To understand the basic concepts of different types of electrical machines and their performance.</li> <li>To know the different methods of starting D.C motors and induction motors.</li> <li>To introduce the controllers for automation</li> </ol>			
<b>Syllabus</b>			
DC Machines, transformers, three phase induction motor, single phase induction motor, stepper motor, controllers for automation.			
<b>Expected outcome .</b>			
The students will be able to			
<ol style="list-style-type: none"> <li>Select a drive for a particular application based on power rating.</li> <li>Select a drive based on mechanical characteristics for a particular drive application.</li> <li>Discuss the controllers used for automation</li> </ol>			
<b>Text Books:</b>			
<ol style="list-style-type: none"> <li>Kothari D. P. and I. J. Nagrath, Electrical Machines, Tata McGraw Hill, 2004.</li> <li>Nagrath .I.J. &amp; Kothari .D.P, Electrical Machines, Tata McGraw-Hill, 1998</li> <li>Richard Crowder, Electrical Drives and Electromechanical systems, Elsevier, 2013</li> <li>Mehta V. K. and R. Mehta, Principles of Electrical and Electronics, S. Chand &amp; Company Ltd., 1996.</li> <li>Theraja B. L. and A. K. Theraja, A Text Book of Electrical Technology, S. Chand &amp; Company Ltd., 2008.</li> <li>Vedam Subrahmaniam, Electric Drives (concepts and applications), Tata McGraw- Hill, 2001</li> </ol>			
<b>References:</b>			
<ol style="list-style-type: none"> <li>H.Partab, Art and Science and Utilisation of electrical energy, Dhanpat Rai and Sons, 1994</li> <li>M. D.Singh, K. B. Khanchandani, Power Electronics, Tata McGraw-Hill, 1998</li> <li>Pillai.S,K A first course on Electric drives, Wiley Eastern Limited, 1998</li> </ol>			
<b>Course Plan</b>			
Module	Contents	Hours	Sem. Exam Marks
I	DC Machines-principle of operation-emf equation-types of excitations. Separately excited, shunt and series excited DC generators, compound generators. General idea of armature reaction, OCC and load characteristics - simple numerical problems.	6	15%
II	Principles of DC motors-torque and speed equations-torque speed characteristics- variations of speed, torque and power with motor current. Applications of dc shunt series and compound motors. Principles of starting, losses and efficiency – load test- simple numerical problems.	6	15%
<b>FIRST INTERNAL EXAMINATION</b>			
III	Transformers – principles of operations – emf equation- vector	7	15%

	diagrams- losses and efficiency – OC and SC tests. Equivalent circuits- efficiency calculations- maximum efficiency – all day efficiency – simple numerical problems. Auto transformers constant voltage transformer- instrument transformers.		
<b>IV</b>	Three phase induction motors- slip ring and squirrel cage types- principles of operation – rotating magnetic field- torque slip characteristics- no load and blocked rotor tests. Circle diagrams- methods of starting – direct online – auto transformer starting	7	15%
<b>SECOND INTERNAL EXAMINATION</b>			
<b>V</b>	Single phase motors- principle of operation of single phase induction motor – split phase motor – capacitor start motor- stepper motor- universal motor Synchronous machines types – emf equation of alternator – regulation of alternator by emf method. Principles of operation of synchronous motors- methods of starting- V curves- synchronous condenser	8	20%
<b>VI</b>	Stepper motors: Principle of operation, multistack variable reluctance motors, single-stack variable reluctance motors, Hybrid stepper motors, Linear stepper motor, comparison, Torque-speed characteristics, control of stepper motors Controllers for automation, servo control, Digital controllers, Advanced control systems, Digital signal processors, motor controllers, Axis controllers, Machine tool controllers, Programmable Logic Controllers	8	20%
<b>END SEMESTER EXAM</b>			

### QUESTION PAPER PATTERN:

**Maximum marks: 100**

**Time: 3 hrs**

The question paper should consist of three parts

**Part A**

There should be 2 questions each from module I and II

Each question carries 10 marks

Students will have to answer any three questions out of 4 (3X10 marks =30 marks)

**Part B**

There should be 2 questions each from module III and IV

Each question carries 10 marks

Students will have to answer any three questions out of 4 (3X10 marks =30 marks)

**Part C**

There should be 3 questions each from module V and VI

Each question carries 10 marks

Students will have to answer any four questions out of 6 (4X10 marks =40 marks)

Note: in all parts each question can have a maximum of four sub questions



## MODULE 1

*DC Machines- principle of operation-emf equation-types of excitations. Separately excited, shunt and series excited DC generators, compound generators. General idea of armature reaction, OCC and load characteristics - simple numerical problems.*

**Introduction**

An electric generator is a machine that converts mechanical energy into electrical energy. An electric generator is based on the principle that whenever flux is cut by a conductor, an e.m.f. is induced which will cause a current to flow if the conductor circuit is closed. The direction of induced e.m.f. (and hence current) is given by Fleming's right hand rule. Therefore, the essential components of a generator are:

- (a) a magnetic field
- (b) conductor or a group of conductors
- (c) motion of conductor w.r.t. magnetic field.

**Simple Loop Generator**

Consider a single turn loop ABCD rotating clockwise in a uniform magnetic field with a constant speed as shown in Fig.

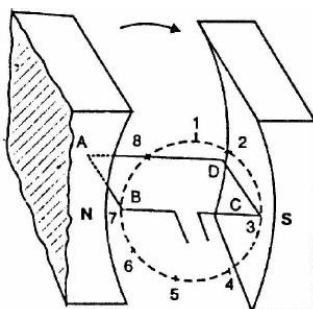


Fig. (1.1)

As the loop rotates, the flux linking the coil sides AB and CD changes continuously. Hence the e.m.f. induced in these coil sides also changes but the e.m.f. induced in one coil side adds to that induced in the other.

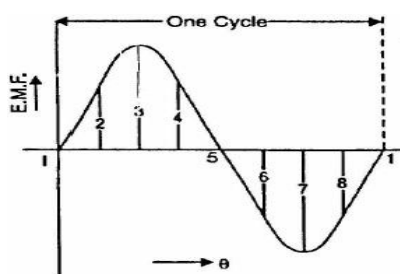


Fig. (1.2)

- (i) When the loop is in position no. 1 [See Fig. 1.1], the generated e.m.f. is zero because the coil sides (AB and CD) are cutting no flux but are moving parallel to it.
- (ii) When the loop is in position no. 2, the coil sides are moving at an angle to the flux and, therefore, a low e.m.f. is generated as indicated by point 2 in Fig. (1.2).
- (iii) When the loop is in position no. 3, the coil sides (AB and CD) are at right angle to the flux and are, therefore, cutting the flux at a maximum rate. Hence at this instant, the generated e.m.f. is maximum as indicated by point 3 in Fig. (1.2).
- (iv) At position 4, the generated e.m.f. is less because the coil sides are cutting the flux at an angle.

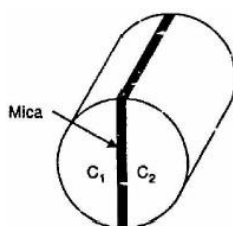
- (v) At position 5, no magnetic lines are cut and hence induced e.m.f. is zero as indicated by point 5 in Fig. (1.2).
- (vi) At position 6, the coil sides move under a pole of opposite polarity and hence the direction of generated e.m.f. is reversed. The maximum e.m.f. in this direction (i.e., reverse direction, See Fig. 1.2) will be when the loop is at position 7 and zero when at position 1. This cycle repeats with each revolution of the coil.

Note that e.m.f. generated in the loop is alternating one. It is because any coil side; say AB has e.m.f. in one direction when under the influence of N-pole and in the other direction when under the influence of S-pole. If a load is connected across the ends of the loop, then alternating current will flow through the load.

The alternating voltage generated in the loop can be converted into direct voltage by a device called commutator. We then have the d.c. generator. In fact, a commutator is a mechanical rectifier.

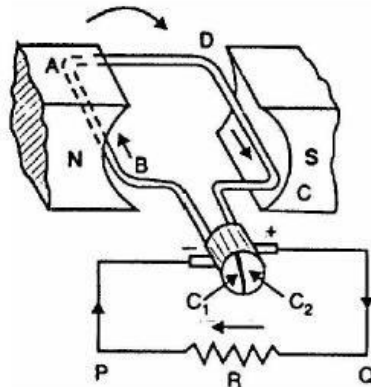
**Commutator**

Connection of the coil side to the external load is reversed at the same instant the current in the coil side reverses, the current through the load will be direct current. This is what a commutator does.



The above shows a commutator having two segments C1 and C2. It consists of a cylindrical metal ring cut into two halves or segments C1 and C2 respectively separated by a thin sheet of mica.

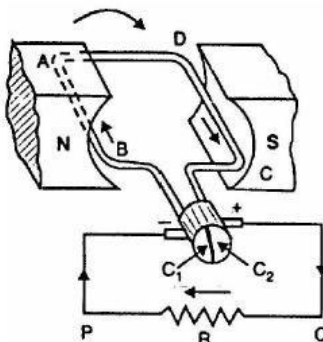
The commutator is mounted on but insulated from the rotor shaft. The ends of coil sides AB and CD are connected to the segments C1 and C2 respectively as shown in Fig.



Two stationary carbon brushes rest on the commutator and lead current to the external load. With this arrangement, the commutator at all times connects the coil side under S-pole to the +ve brush and that under N-pole to the -ve brush.

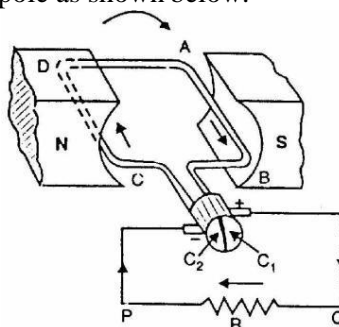
**DC Generator Working**

- (i) In the below Fig., the coil sides AB and CD are under N-pole and S-pole respectively.



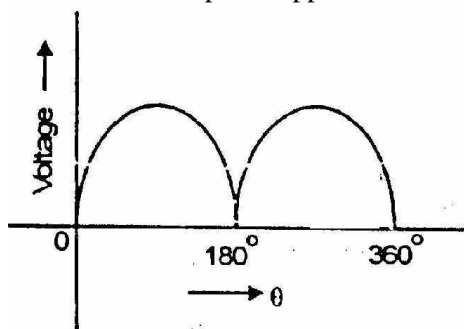
The segment  $C_1$  connects the coil side AB to point P of the load resistance R and the segment  $C_2$  connects the coil side CD to point Q of the load. Also note the direction of current through load. It is from Q to P.

- (ii) After half a revolution of the loop (i.e.,  $180^\circ$  rotation), the coil side AB is under S-pole and the coil side CD under N-pole as shown below.



The currents in the coil sides now flow in the reverse direction but the segments  $C_1$  and  $C_2$  have also moved through  $180^\circ$  i.e., segment  $C_1$  is now in contact with +ve brush and segment  $C_2$  in contact with -ve brush. Note that commutator has reversed the coil connections to the load i.e., coil side AB is now connected to point Q of the load and coil side CD to the point P of the load. Also note the direction of current through the load. It is again from Q to P.

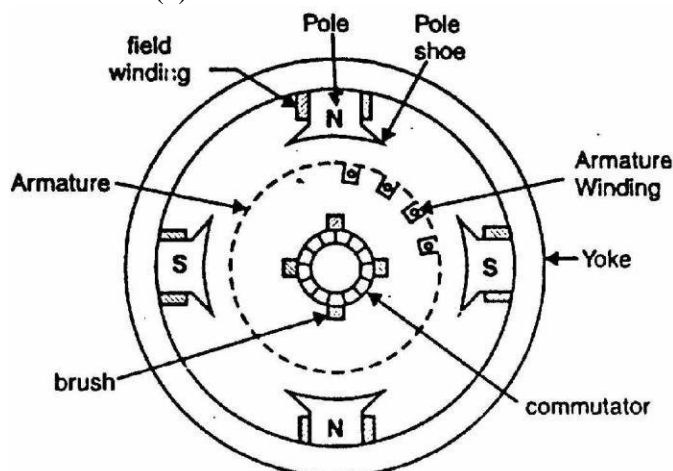
Thus the alternating voltage generated in the loop will appear as direct voltage across the brushes.



The e.m.f. generated in the armature winding of a d.c. generator is alternating one. By the use of commutator we convert the generated alternating e.m.f. into direct voltage. The purpose of brushes is simply to lead current from the rotating loop or winding to the external stationary load.

**Construction of d.c. Generator**

All d.c. machines have five principal components viz., (i) field system (ii) armature core (iii) armature winding (iv) commutator (v) brushes



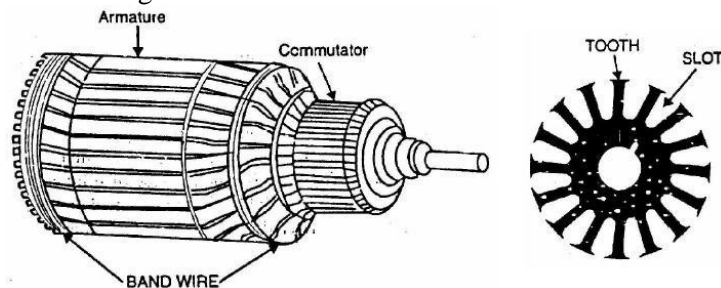
**(i) Field system**

The function of the field system is to produce uniform magnetic field within which the armature rotates. It consists of an even number of salient poles bolted to the inside of circular frame

(generally called yoke). The yoke is usually made of solid cast steel whereas the pole pieces are composed of stacked laminations. Field coils are mounted on the poles and carry the d.c. exciting current. The field coils are connected in such a way that adjacent poles have opposite polarity.

### (ii) Armature core

The armature core is keyed to the machine shaft and rotates between the field poles. It consists of slotted soft-iron laminations (about 0.4 to 0.6 mm thick) that are stacked to form a cylindrical core as shown in Fig.



The laminations are individually coated with a thin insulating film so that they do not come in electrical contact with each other. The purpose of laminating the core is to reduce the eddy current loss. The laminations are slotted to accommodate and provide mechanical security to the armature winding and to give shorter air gap for the flux to cross between the pole face and the armature-teeth.

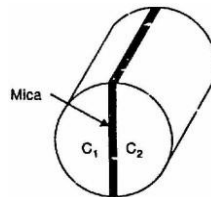
### (iii) Armature winding

The slots of the armature core hold insulated conductors that are connected in a suitable manner. This is known as armature winding. This is the winding in which working e.m.f. is induced.

The armature conductors are connected in series-parallel; the conductors being connected in series so as to increase the voltage and in parallel paths so as to increase the current. The armature winding of a d.c. machine is a closed-circuit winding; the conductors being connected in a symmetrical manner forming a closed loop or series of closed loops.

### (iv) Commutator

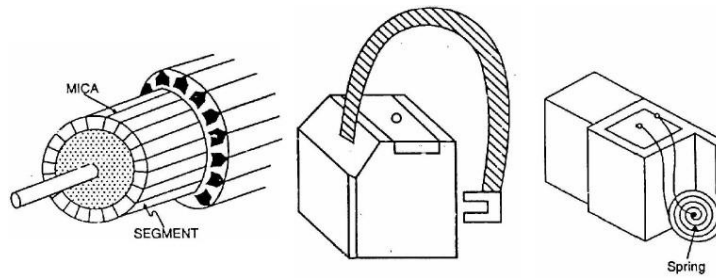
A commutator is a mechanical rectifier which converts the alternating voltage generated in the armature winding into direct voltage across the brushes. The commutator is made of copper segments insulated from each other by mica sheets and mounted on the shaft of the machine.



The armature conductors are soldered to the commutator segments in a suitable manner to give rise to the armature winding. Depending upon the manner in which the armature conductors are connected to the commutator segments, there are two types of armature winding in a d.c. machine viz., (a) lap winding (b) wave winding.

### (v) Brushes

The purpose of brushes is to ensure electrical connections between the rotating commutator and stationary external load circuit. The brushes are made of carbon and rest on the commutator. The brush pressure is adjusted by means of adjustable springs.



If the brush pressure is very large, the friction produces heating of the commutator and the brushes. On the other hand, if it is too weak, the imperfect contact with the commutator may produce sparking. Brushes having the same polarity are connected together so that we have two terminals viz., the +ve terminal and the -ve terminal.

### E.M.F. Equation of a D.C. Generator

Let

- $\phi$  = flux/pole in Wb
- $Z$  = total number of armature conductors
- $P$  = number of poles
- $A$  = number of parallel paths = 2 ... for wave winding  
=  $P$  ... for lap winding
- $N$  = speed of armature in r.p.m.
- $E_g$  = e.m.f. of the generator = e.m.f./parallel path

Flux cut by one conductor in one revolution of the armature,  
 $d\phi = P\phi$  webers

Time taken to complete one revolution,  
 $dt = 60/N$  second

e.m.f generated/conductor =  $\frac{d\phi}{dt} = \frac{P\phi}{60/N} = \frac{P\phi N}{60}$  volts

e.m.f. of generator,  
 $E_g =$  e.m.f. per parallel path  
 $=$  (e.m.f/conductor)  $\times$  No. of conductors in series per parallel path  
 $= \frac{P\phi N}{60} \times \frac{Z}{A}$

$\therefore E_g = \frac{P\phi ZN}{60 A}$

where  $A = 2$  for-wave winding  
 $= P$  for lap winding

### Armature Resistance ( $R_a$ )

The resistance offered by the armature circuit is known as armature resistance ( $R_a$ ) and includes:

- (i) resistance of armature winding
- (ii) resistance of brushes

The armature resistance depends upon the construction of machine. Except for small machines, its value is generally less than  $1\Omega$ .

### TYPES OF D.C. GENERATORS

The magnetic field in a d.c. generator is normally produced by electromagnets rather than permanent magnets. Generators are generally classified according to their methods of field excitation. On this basis, d.c. generators are divided into the following two classes:

- (i) Separately excited d.c. generators
- (ii) Self-excited d.c. generators

The behaviour of a d.c. generator on load depends upon the method of field excitation adopted.

#### (i) Separately Excited D.C. Generators

A d.c. generator whose field magnet winding is supplied from an independent external d.c. source (e.g., a battery etc.) is called a separately excited generator.



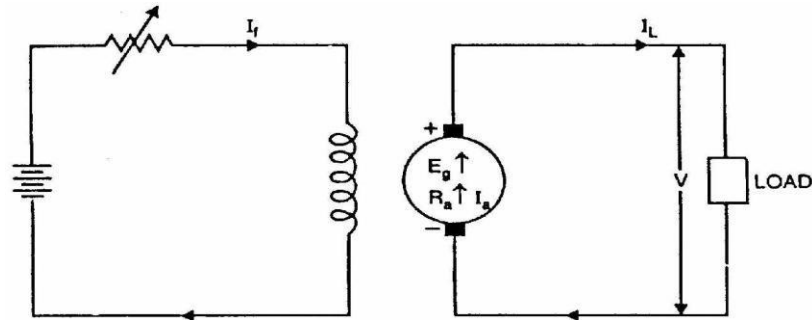


Fig. shows the connections of a separately excited generator.

The voltage output depends upon the speed of rotation of **armature and the field current**

( $E_g = P\phi ZN/60 A$ ). ( $P, Z, 60$  and  $A$  are constants)

**The greater the speed (N) and field current ( $\phi$  is directly proportional to  $I_f$ ), greater is the generated e.m.f.**

It may be noted that separately excited d.c. generators are rarely used in practice. The d.c. generators are normally of self-excited type.

Armature current,  $I_a = I_L$

Terminal voltage,  $V = E_g - I_a R_a$

Electric power developed =  $E_g I_a$

Power delivered to load =  $E_g I_a - I_a^2 R_a = I_a (E_g - I_a R_a) = V I_a$

$I_a^2 R_a$  – armature copper loss

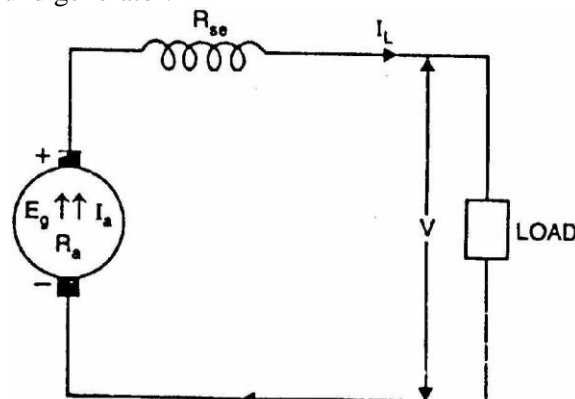
## (ii) Self-Excited D.C. Generators

A d.c. generator whose field magnet winding is supplied current from the output of the generator itself is called a self-excited generator. There are three types of self-excited generators depending upon the manner in which the field winding is connected to the armature, namely;

- (1) Series generator
- (2) Shunt generator
- (3) Compound generator

### (1) Series generator

In a series wound generator, the field winding is connected in series with armature winding so that whole armature current flows through the field winding as well as the load. Fig shows the connections of a series wound generator.



Since the field winding carries the whole of load current, it has a few turns of thick wire having low resistance. Series generators are rarely used except for special purposes e.g., as boosters.

Armature current,  $I_a = I_{se} = I_L = I$  (say)

Terminal voltage,  $V = E_g - I(R_a + R_{se})$

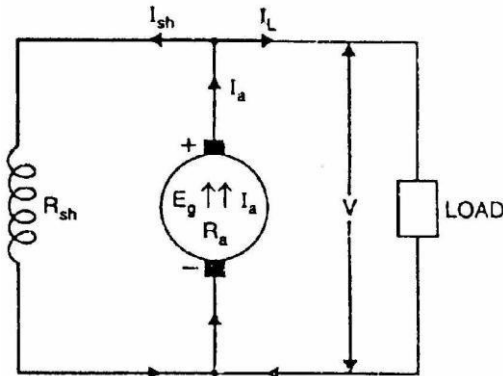
Power developed in armature =  $E_g I_a$

Power delivered to load

$$= E_g I_a - I_a^2 (R_a + R_{se}) = I_a [E_g - I_a (R_a + R_{se})] = V I_a \text{ or } V I_L$$

## (2) Shunt generator

In a shunt generator, the field winding is connected in parallel with the armature winding so that terminal voltage of the generator is applied across it. The shunt field winding has many turns of fine wire having high resistance. Therefore, only a part of armature current flows through shunt field winding and the rest flows through the load. Fig. shows the connections of a shunt-wound generator.



Shunt field current,  $I_{sh} = V/R_{sh}$

Armature current,  $I_a = I_L + I_{sh}$

Terminal voltage,  $V = E_g - I_a R_a$

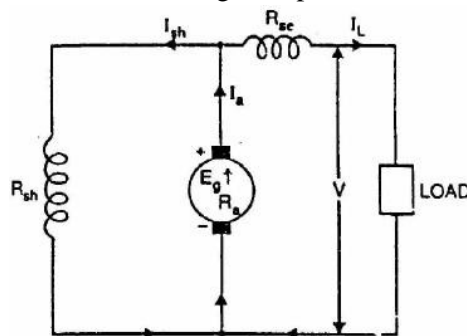
Power developed in armature =  $E_g I_a$

Power delivered to load =  $V I_L$

## (3) Compound generator

In a compound-wound generator, there are two sets of field windings on each pole—one is in series and the other in parallel with the armature. A compound wound generator may be:

(a) **Short Shunt** in which only shunt field winding is in parallel with the armature winding



Series field current,  $I_{se} = I_L$

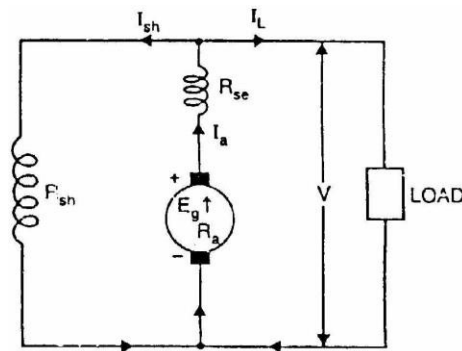
Shunt field current,  $I_{sh} = \frac{V + I_{se} R_{se}}{R_{sh}}$

Terminal voltage,  $V = E_g - I_a R_a - I_{se} R_{se}$

Power developed in armature =  $E_g I_a$

Power delivered to load =  $V I_L$

(b) **Long Shunt** in which shunt field winding is in parallel with both series field and armature winding



Series field current,  $I_{se} = I_a = I_L + I_{sh}$

Shunt field current,  $I_{sh} = V/R_{sh}$

Terminal voltage,  $V = E_g - I_a(R_a + R_{se})$

Power developed in armature =  $E_g I_a$

Power delivered to load =  $V I_L$

### Brush Contact Drop

It is the voltage drop over the brush contact resistance when current flows. Obviously, its value will depend upon the amount of current flowing and the value of contact resistance. This drop is generally small.

### ARMATURE REACTION

In a d.c. generator, the purpose of field winding is to produce magnetic field (called main flux) whereas the purpose of armature winding is to carry armature current.

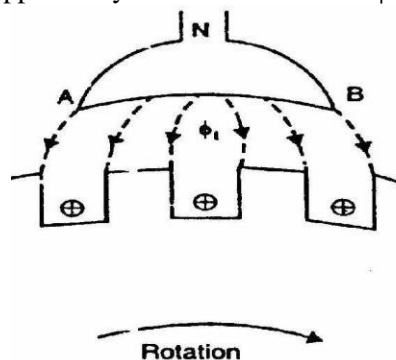
Although the armature winding is not provided for the purpose of producing a magnetic field, nevertheless the current in the armature winding will also produce magnetic flux (called armature flux).

**The armature flux distorts and weakens the main flux posing problems for the proper operation of the d.c. generator. The action of armature flux on the main flux is called armature reaction.**

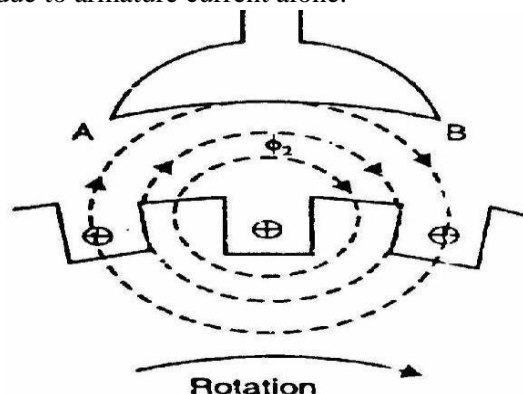
*(The only flux acting in a d.c. machine is that due to the main poles called main flux. However, current flowing through armature conductors also creates a magnetic flux (called armature flux) that distorts and weakens the flux coming from the poles. This distortion and field weakening takes place in both generators and motors. The action of armature flux on the main flux is known as armature reaction.)*

### Explanation

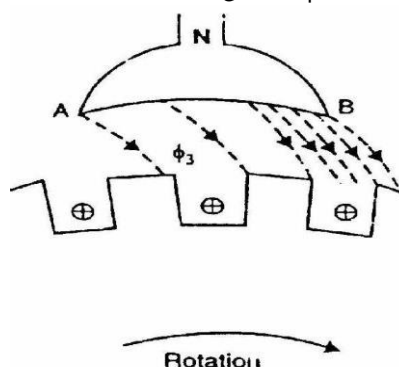
Consider one pole of the generator. When the generator is on no-load, a small current flowing in the armature conductors does not appreciably affect the main flux  $\phi_1$  coming from the pole.



When the generator is loaded, the current flowing through armature conductors sets up flux  $\phi_2$ . The Fig. below shows flux due to armature current alone.



By superimposing  $\phi_1$  and  $\phi_2$ , we obtain the resulting flux  $\phi_3$  as shown in Fig.



From the Fig. it is clear that flux density at the trailing pole tip (point B) is increased while at the leading pole tip (point A) it is decreased. This unequal field distribution produces the following two effects:

1. It demagnetizes or weakens the main flux.
2. It cross-magnetizes or distorts the main flux.

## D.C. GENERATOR CHARACTERISTICS

### 1. Open Circuit Characteristic (O.C.C.)

This curve shows the relation between the generated e.m.f. at no-load ( $E_0$ ) and the field current ( $I_f$ ) at constant speed. It is also known as magnetic characteristic or no-load saturation curve. Its shape is practically the same for all generators whether separately or self-excited. The data for O.C.C. curve are obtained experimentally by operating the generator at no load and constant speed and recording the change in terminal voltage as the field current is varied.

### 2. Internal or Total characteristic ( $E/I_a$ )

This curve shows the relation between the generated e.m.f. on load ( $E$ ) and the armature current ( $I_a$ ).

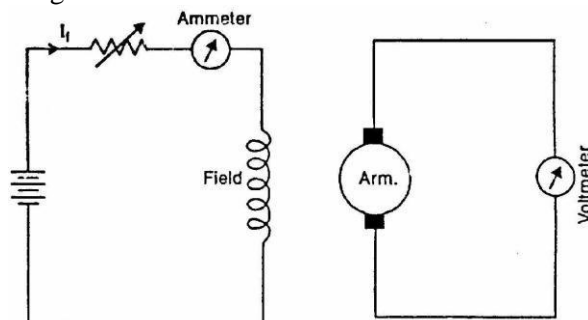
The e.m.f.  $E$  is less than  $E_0$  due to the demagnetizing effect of armature reaction. Therefore, this curve will lie below the open circuit characteristic (O.C.C.). The internal characteristic can be obtained from external characteristic if winding resistances are known because armature reaction effect is included in both characteristics.

### 3. External characteristic ( $V/I_L$ )

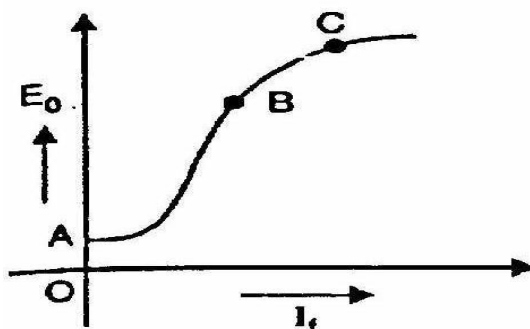
This curve shows the relation between the terminal voltage ( $V$ ) and load current ( $I_L$ ). The terminal voltage  $V$  will be less than  $E$  due to voltage drop in the armature circuit. Therefore, this curve will lie below the internal characteristic. This characteristic is very important in determining the suitability of a generator for a given purpose. It can be obtained by making simultaneous measurements of terminal voltage and load current (with voltmeter and ammeter) of a loaded generator.

### Open Circuit Characteristic of a self excited D.C. Generator

The O.C.C. for a d.c. generator is determined as follows. The field winding of the d.c. generator (series or shunt) is disconnected from the machine and is separately excited from an external d.c. source as shown in Fig.



The generator is run at fixed speed (i.e., normal speed). The field current ( $I_f$ ) is increased from zero in steps and the corresponding values of generated e.m.f. ( $E_0$ ) read off on a voltmeter connected across the armature terminals. On plotting the relation between  $E_0$  and  $I_f$ , we get the open circuit characteristic as shown in Fig.



The following points may be noted from O.C.C.:

- (i) When the field current is zero, there is some generated e.m.f.  $OA$ . This is due to the residual magnetism in the field poles.
- (ii) Over a fairly wide range of field current (upto point B in the curve), the curve is linear. It is because in this range, reluctance of iron is negligible as compared with that of air gap. The air gap reluctance is constant and hence linear relationship.
- (iii) After point B on the curve, the reluctance of iron also comes into picture. It is because at higher flux densities,  $\mu_r$  for iron decreases and reluctance of iron is no longer negligible. Consequently, the curve deviates from linear relationship.
- (iv) After point C on the curve, the magnetic saturation of poles begins and  $E_0$  tends to level off. The reader may note that the O.C.C. of even self-excited generator is obtained by running it as a separately excited generator.

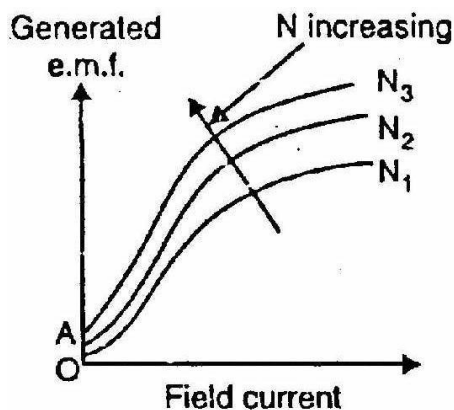
### Characteristics of a Separately Excited D.C. Generator

The obvious disadvantage of a separately excited d.c. generator is that we require an external d.c. source for excitation. But since the output voltage may be controlled more easily and over a wide range (from zero to a maximum), this type of excitation finds many applications.

#### (i) Open circuit characteristic.

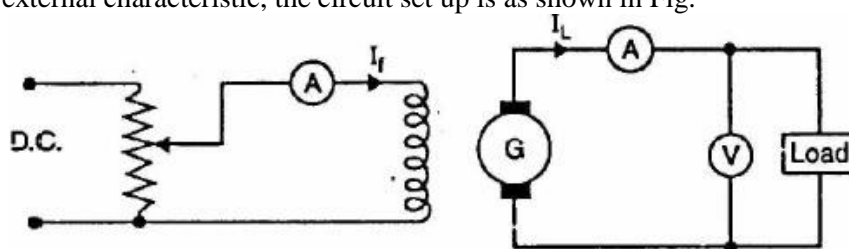
The O.C.C. of a separately excited generator is determined in a manner described in previous section. Fig. shows the variation of generated e.m.f. on no load with field current for various fixed speeds. Note that if the value of constant speed is increased, the steepness of the curve also increases. When the field current is zero, the residual magnetism in the poles will give rise to the small initial e.m.f. as shown.





**(ii) Internal and External Characteristics**

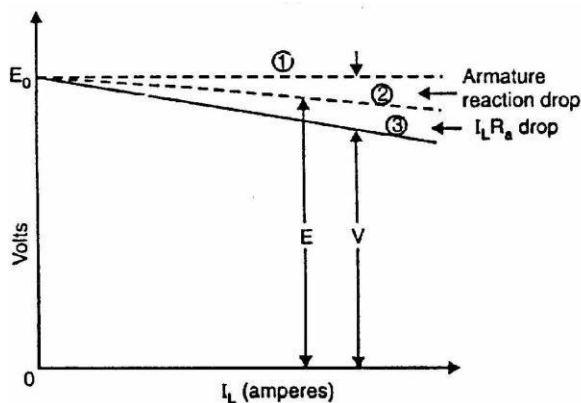
The external characteristic of a separately excited generator is the curve between the terminal voltage ( $V$ ) and the load current  $I_L$  (which is the same as armature current in this case). In order to determine the external characteristic, the circuit set up is as shown in Fig.



As the load current increases, the terminal voltage falls due to two reasons:

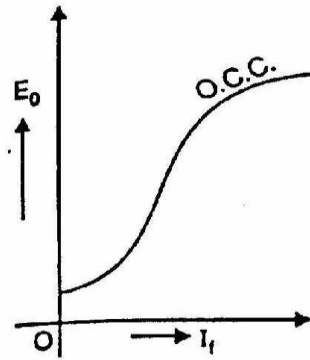
- (a) The armature reaction weakens the main flux so that actual e.m.f. generated  $E$  on load is less than that generated ( $E_0$ ) on no load.
- (b) There is voltage drop across armature resistance ( $= I_L R_a = I_a R_a$ ). Due to these reasons, the external characteristic is a drooping curve [curve 3 in Fig.]. Note that in the absence of armature reaction and armature drop, the generated e.m.f. would have been  $E_0$  (curve 1).

The internal characteristic can be determined from external characteristic by adding  $I_L R_a$  drop to the external characteristic. It is because armature reaction drop is included in the external characteristic. Curve 2 is the internal characteristic of the generator and should obviously lie above the external characteristic.

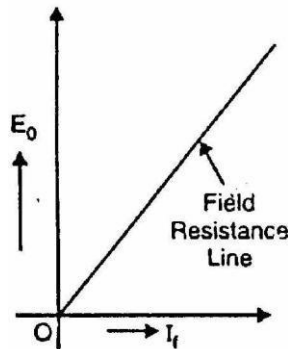


**Voltage Build-Up in a Self-Excited Generator**

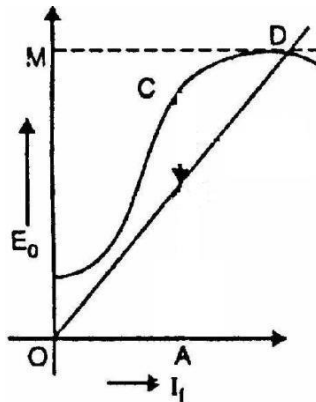
Consider a shunt generator. If the generator is run at a constant speed, some e.m.f. will be generated due to residual magnetism in the main poles. This small e.m.f. circulates a field current which in turn produces additional flux to reinforce the original residual flux (provided field winding connections are correct). This process continues and the generator builds up the normal generated voltage following the O.C.C. shown in Fig.



The field resistance  $R_f$  can be represented by a straight line passing through the origin as shown in Fig.



The voltage build up of the generator is given by the point of intersection of O.C.C. and field resistance line.



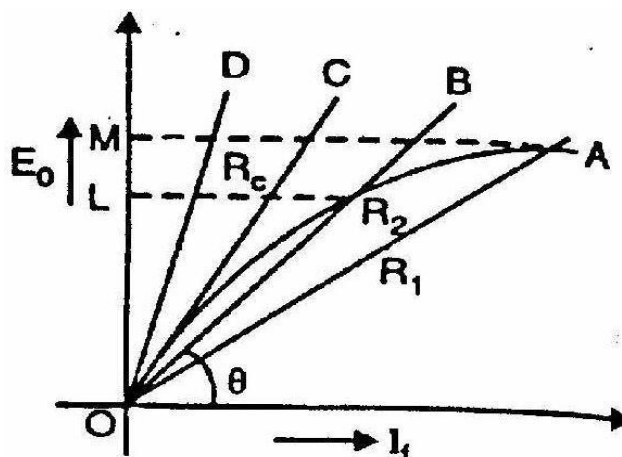
Thus in Fig., D is point of intersection of the two curves. Hence the generator will build up a voltage OM.

**Critical Field Resistance for a Shunt Generator**

The voltage build up in a shunt generator depends upon field circuit resistance. If the field circuit resistance is  $R_1$  (line OA), then generator will build up a voltage OM as shown in Fig.

If the field circuit resistance is increased to  $R_2$  (line OB), the generator will build up a voltage OL, slightly less than OM.

As the field circuit resistance is increased, the slope of resistance line also increases. When the field resistance line becomes tangent (line OC) to O.C.C., the generator would just excite.



If the field circuit resistance is increased beyond this point (say line OD), the generator will fail to excite. The field circuit resistance represented by line OC (tangent to O.C.C.) is called critical field resistance RC for the shunt generator.

*The maximum field circuit resistance (for a given speed) with which the shunt generator would just excite is known as its critical field resistance.*

**Drawing of OCC at different speed**

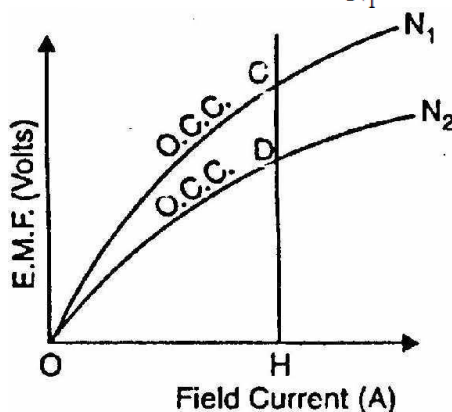
The given O.C.C. of a generator at a constant speed  $N_1$ , then we can easily draw the O.C.C. at any other constant speed  $N_2$ .

Here we are given O.C.C. at a constant speed  $N_1$ . It is desired to find the O.C.C. at constant speed  $N_2$  (it is assumed that  $N_1 < N_2$ ).

For constant excitation,  $E \propto N$ .

$$\therefore \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

or  $E_2 = E_1 \times \frac{N_2}{N_1}$



From the above Fig. , for a particular  $I_f = OH$ ,

$$E_1 = HC.$$

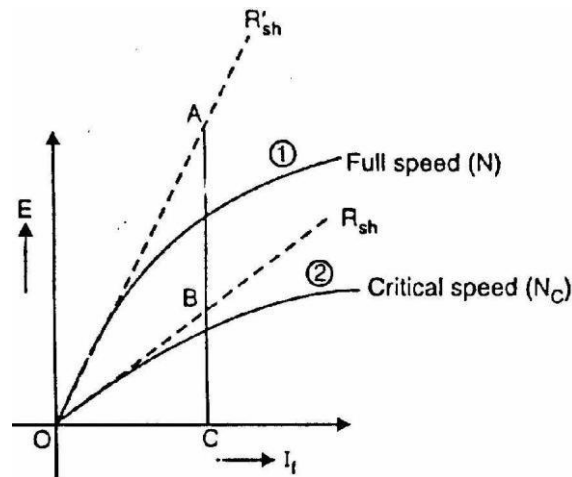
Therefore, the new value of e.m.f. ( $E_2$ ) for the same  $I_f$  but at  $N_2$  is

$$E_2 = HC \times \frac{N_2}{N_1} = HD$$

This locates the point D on the new O.C.C. at  $N_2$ . Similarly, other points can be located taking different values of  $I_f$ . The locus of these points will be the O.C.C. at  $N_2$ .

**Critical Speed (NC)**

The critical speed of a shunt generator is the minimum speed below which it fails to excite. Clearly, it is the speed for which the given shunt field resistance represents the critical resistance.



In Fig. , curve 2 corresponds to critical speed because the shunt field resistance ( $R_{sh}$ ) line is tangential to it.

If the generator runs at full speed  $N$ , the new O.C.C. moves upward and the  $R'_{sh}$  line represents critical resistance for this speed.

$$\therefore \text{Speed} \propto \text{Critical resistance}$$

In order to find critical speed, take any convenient point  $C$  on excitation axis and erect a perpendicular so as to cut  $R_{sh}$  and  $R'_{sh}$  lines at points  $B$  and  $A$  respectively. Then,

$$\frac{BC}{AC} = \frac{N_C}{N}$$

$$\text{or } N_C = N \times \frac{BC}{AC}$$

### Conditions for Voltage Build-Up of a Shunt Generator

The necessary conditions for voltage build-up in a shunt generator are:

- (i) There must be some residual magnetism in generator poles.
- (ii) The connections of the field winding should be such that the field current strengthens the residual magnetism.
- (iii) The resistance of the field circuit should be less than the critical resistance. In other words, the speed of the generator should be higher than the critical speed.

## MODULE 2

*Principles of DC motors-torque and speed equations-torque speed characteristics- variations of speed, torque and power with motor current. Applications of dc shunt series and compound motors. Principles of starting, losses and efficiency – load test- simple numerical problems.*

**D.C. Motor Principle**

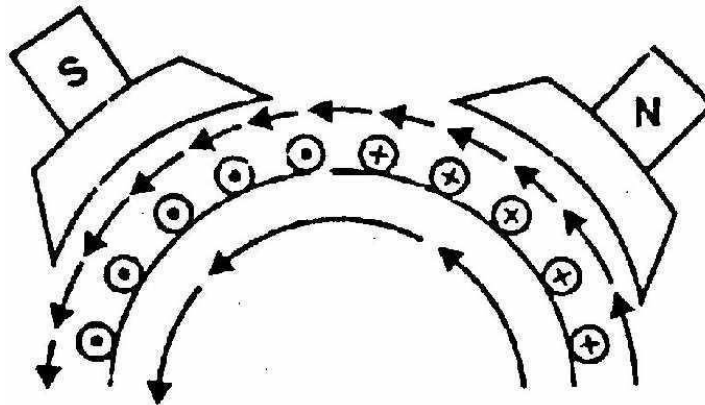
A machine that converts d.c. power into mechanical power is known as a d.c. motor. Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by Fleming's left hand rule and magnitude is given by;

$$F = BI\ell \text{ newtons}$$

Basically, there is no constructional difference between a d.c. motor and a d.c. generator. The same d.c. machine can be run as a generator or motor.

**Working of D.C. Motor**

Consider a part of a multipolar d.c. motor as shown in Fig.



When the terminals of the motor are connected to an external source of d.c. supply:

- (i) the field magnets are excited developing alternate N and S poles;
- (ii) the armature conductors carry currents.

All conductors under N-pole carry currents in one direction while all the conductors under S-pole carry currents in the opposite direction.

Suppose the conductors under N-pole carry currents into the plane of the paper and those under S-pole carry currents out of the plane of the paper as shown in Fig.

Since each armature conductor is carrying current and is placed in the magnetic field, mechanical force acts on it. Referring to Fig. and applying Fleming's left hand rule, it is clear that force on each conductor is tending to rotate the armature in anticlockwise direction.

All these forces add together to produce a driving torque which sets the armature rotating.

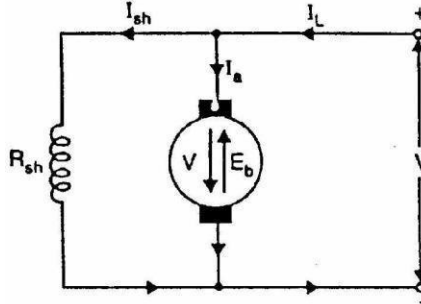
When the conductor moves from one side of a brush to the other, the current in that conductor is reversed and at the same time it comes under the influence of next pole which is of opposite polarity. Consequently, the direction of force on the conductor remains the same.

**Back or Counter E.M.F.**

When the armature of a d.c. motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence e.m.f. is induced in them as in a generator. The induced e.m.f. acts in opposite direction to the applied voltage  $V$  (Lenz's law) and is known as back or counter e.m.f.  $E_b$ . The back e.m.f.  $E_b (= P \phi ZN/60 A)$  is always less than the applied voltage  $V$ , although this difference is small when the motor is running under normal conditions.



Consider a shunt wound motor shown in Fig.



When d.c. voltage  $V$  is applied across the motor terminals, the field magnets are excited and armature conductors are supplied with current. Therefore, driving torque acts on the armature which begins to rotate. As the armature rotates, back e.m.f.  $E_b$  is induced which opposes the applied voltage  $V$ . The applied voltage  $V$  has to force current through the armature against the back e.m.f.  $E_b$ . The electric work done in overcoming and causing the current to flow against  $E_b$  is converted into mechanical energy developed in the armature. It follows, therefore, that energy conversion in a d.c. motor is only possible due to the production of back e.m.f.  $E_b$ .

Net voltage across armature circuit =  $V - E_b$

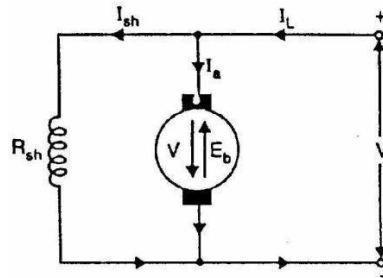
If  $R_a$  is the armature circuit resistance, then,

$$I_a = \frac{V - E_b}{R_a}$$

Since  $V$  and  $R_a$  are usually fixed, the value of  $E_b$  will determine the current drawn by the motor. If the speed of the motor is high, then back e.m.f.  $E_b (= P \phi ZN/60 A)$  is large and hence the motor will draw less armature current and vice versa.

**Voltage Equation of D.C. Motor**

Let in a d.c. motor



$V$  = applied voltage

$R_a$  = armature resistance

$E_b$  = back e.m.f.

$I_a$  = armature current

Since back e.m.f.  $E_b$  acts in opposition to the applied voltage  $V$ , the net voltage across the armature circuit is  $V - E_b$ . The armature current  $I_a$  is given by;

$$I_a = \frac{V - E_b}{R_a}$$

$$V = E_b + I_a R_a$$

This is known as voltage equation of the d.c. motor.

**Power Equation**

The above voltage is multiplied by  $I_a$  throughout, we get,

$$VI_a = E_b I_a + I_a^2 R_a$$

This is known as power equation of the d.c. motor.

$VI_a$  = electric power supplied to armature (armature input)

$E_b I_a$  = power developed by armature (armature output)

$I_a^2 R_a$  = electric power wasted in armature (armature Cu loss)

Thus out of the armature input, a small portion (about 5%) is wasted as  $I_a^2 R_a$  and the remaining portion  $E_b I_a$  is converted into mechanical power within the armature.

### Condition for Maximum Power

The mechanical power developed by the motor is  $P_m = E_b I_a$

$$\text{Now } P_m = VI_a - I_a^2 R_a$$

Since,  $V$  and  $R_a$  are fixed, power developed by the motor depends upon armature current. For maximum power,  $dP_m/dI_a$  should be zero.

$$\therefore \frac{dP_m}{dI_a} = V - 2I_a R_a = 0$$

$$\text{or } I_a R_a = \frac{V}{2}$$

$$\text{Now, } V = E_b + I_a R_a = E_b + \frac{V}{2} \quad \left[ \because I_a R_a = \frac{V}{2} \right]$$

$$\therefore E_b = \frac{V}{2}$$

Hence mechanical power developed by the motor is maximum when back e.m.f. is equal to half the applied voltage.

### Limitations

In practice, we never aim at achieving maximum power due to the following reasons:

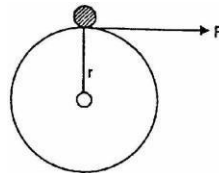
- (i) The armature current under this condition is very large—much excess of rated current of the machine.
- (ii) Half of the input power is wasted in the armature circuit. In fact, if we take into account other losses (iron and mechanical), the efficiency will be well below 50%.

### ARMATURE TORQUE OF D.C. MOTOR

Torque is the turning moment of a force about an axis and is measured by the product of force ( $F$ ) and radius ( $r$ ) at right angle to which the force acts i.e.

$$T = F \times r$$

In a d.c. motor, each conductor is acted upon by a circumferential force  $F$  at a distance  $r$ , the radius of the armature. Therefore, each conductor exerts a torque, tending to rotate the armature.



The sum of the torques due to all armature conductors is known as gross or armature torque ( $T_a$ ).

Let in a d.c. motor

$r$  = average radius of armature in m

$\square$  = effective length of each conductor in m

$Z$  = total number of armature conductors

$A$  = number of parallel paths

$i$  = current in each conductor =  $I_a/A$

$B$  = average flux density in  $\text{Wb/m}^2$

$\phi$  = flux per pole in Wb

$P$  = number of poles

Force on each conductor,  $F = B i \square$  newtons

Torque due to one conductor =  $F \times r$  newton- metre

Total armature torque,  $T_a = Z F r$  newton-metre

$$= Z B i \square r$$

Now  $i = I_a/A$ ,  $B = \phi/a$  where  $a$  is the x-sectional area of flux path per pole at radius  $r$ .

Clearly,  $a = 2\pi r \square /P$ .

$$\begin{aligned}\therefore T_a &= Z \times \left(\frac{\phi}{2}\right) \times \left(\frac{I_a}{A}\right) \times \ell \times r \\ &= Z \times \frac{\phi}{2\pi r \ell / P} \times \frac{I_a}{A} \times \ell \times r = \frac{Z\phi I_a P}{2\pi A} \text{ N - m}\end{aligned}$$

$$\text{or } T_a = 0.159 Z\phi I_a \left(\frac{P}{A}\right) \text{ N - m}$$

Since Z, P and A are fixed for a given machine,

$$\therefore T_a \propto \phi I_a$$

Hence torque in a d.c. motor is directly proportional to flux per pole and armature current.

(i) For a shunt motor, flux  $\phi$  is practically constant.

$$\therefore T_a \propto I_a$$

(ii) For a series motor, flux  $\phi$  is directly proportional to armature current  $I_a$  provided magnetic saturation does not take place.

$$\therefore T_a \propto I_a^2$$

Up to magnetic saturation.

#### Alternative expression for $T_a$

$$E_b = \frac{P\phi ZN}{60A}$$

$$\therefore \frac{P\phi Z}{A} = \frac{60 \times E_b}{N}$$

From Eq.(i), we get the expression of  $T_a$  as:

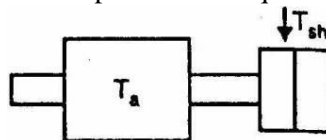
$$T_a = 0.159 \times \left(\frac{60 \times E_b}{N}\right) \times I_a$$

$$\text{or } T_a = 9.55 \times \frac{E_b I_a}{N} \text{ N - m}$$

Note that developed torque or gross torque means armature torque  $T_a$ .

#### Shaft Torque ( $T_{sh}$ )

The torque which is available at the motor shaft for doing useful work is known as shaft torque. It is represented by  $T_{sh}$ . Fig. illustrates the concept of shaft torque.



The total or gross torque  $T_a$  developed in the armature of a motor is not available at the shaft because a part of it is lost in overcoming the iron and frictional losses in the motor. Therefore, shaft torque  $T_{sh}$  is somewhat less than the armature torque  $T_a$ . The difference  $T_a - T_{sh}$  is called lost torque.

$$T_a - T_{sh} = 9.55 \times \frac{\text{Iron and frictional losses}}{N}$$

As stated above, it is the shaft torque  $T_{sh}$  that produces the useful output. If the speed of the motor is  $N$  r.p.m., then,

$$\text{Output in watts} = \frac{2\pi N T_{sh}}{60}$$

$$\text{or } T_{sh} = \frac{\text{Output in watts}}{2\pi N/60} \text{ N - m}$$

$$\text{or } T_{sh} = 9.55 \times \frac{\text{Output in watts}}{N} \text{ N - m} \quad \left( \because \frac{60}{2\pi} = 9.55 \right)$$

### Brake Horse Power (B.H.P.)

The horse power developed by the shaft torque is known as brake horsepower (B.H.P.). If the motor is running at  $N$  r.p.m. and the shaft torque is  $T_{sh}$  newton-metres, then,

$$\begin{aligned} \text{W.D./revolution} &= \text{force} \times \text{distance moved in 1 revolution} \\ &= F \times 2\pi r = 2\pi \times T_{sh} \text{ J} \end{aligned}$$

$$\text{W.D./minute} = 2\pi N T_{sh} \text{ J}$$

$$\text{W.D./sec.} = \frac{2\pi N T_{sh}}{60} \text{ Js}^{-1} \text{ or watts} = \frac{2\pi N T_{sh}}{60 \times 746} \text{ H.P.}$$

$$\therefore \text{Useful output power} = \frac{2\pi N T_{sh}}{60 \times 746} \text{ H.P.}$$

$$\text{or } \text{B.H.P.} = \frac{2\pi N T_{sh}}{60 \times 746}$$

### Speed of a D.C. Motor

$$E_b = V - I_a R_a$$

$$\text{But } E_b = \frac{P\phi ZN}{60 A}$$

$$\therefore \frac{P\phi ZN}{60 A} = V - I_a R_a$$

$$\text{or } N = \frac{(V - I_a R_a) 60 A}{\phi P Z}$$

$$\text{or } N = K \frac{(V - I_a R_a)}{\phi} \quad \text{where } K = \frac{60 A}{P Z}$$

$$\text{But } V - I_a R_a = E_a$$

$$\therefore N = K \frac{E_b}{\phi}$$

$$\text{or } N \propto \frac{E_b}{\phi}$$

Therefore, in a d.c. motor, speed is directly proportional to back e.m.f.  $E_b$  and inversely proportional to flux per pole  $\phi$ .

### Speed Relations

If a d.c. motor has initial values of speed, flux per pole and back e.m.f. as  $N_1$ ,  $\phi_1$  and  $E_{b1}$  respectively and the corresponding final values are  $N_2$ ,  $\phi_2$  and  $E_{b2}$ , then,

$$N_1 \propto \frac{E_{b1}}{\phi_1} \quad \text{and} \quad N_2 \propto \frac{E_{b2}}{\phi_2}$$

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

(i) For a shunt motor, flux practically remains constant so that  $\phi_1 = \phi_2$ .

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

(ii) For a series motor,  $\phi \propto I_a$  prior to saturation.

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

where  $I_{a1}$  = initial armature current  
 $I_{a2}$  = final armature current

### Speed Regulation

The speed regulation of a motor is the change in speed from full-load to no-load and is expressed as a percentage of the speed at full-load i.e.

$$\% \text{ Speed regulation} = \frac{\text{N.L. speed} - \text{F.L. speed}}{\text{F.L. speed}} \times 100$$

$$= \frac{N_0 - N}{N} \times 100$$

where  $N_0$  = No - load speed  
 $N$  = Full - load speed

### Torque and Speed of a D.C. Motor

For any motor, the torque and speed are very important factors. When the torque increases, the speed of a motor increases and vice-versa. We have seen that for a d.c. motor;

$$N = K \frac{(V - I_a R_a)}{\phi} = \frac{K E_b}{\phi} \quad \text{(i)}$$

$$T_a \propto \phi I_a \quad \text{(ii)}$$

If the flux decreases, from Eq.(i), the motor speed increases but from Eq.(ii) the motor torque decreases.

This is not possible because the increase in motor speed must be the result of increased torque. Indeed, it is so in this case. When the flux decreases slightly, the armature current increases to a large value. As a result, in spite of the weakened field, the torque is momentarily increased to a high value and will exceed considerably the value corresponding to the load. The surplus torque available causes the motor to accelerate and back e.m.f. ( $E_a = P \phi Z N/60A$ ) to rise. Steady conditions of speed will ultimately be achieved when back e.m.f. has risen to such a value that armature current [ $I_a = (V - E_a)/R_a$ ] develops torque just sufficient to drive the load.

### D.C. MOTOR CHARACTERISTICS

There are three principal types of d.c. motors viz., shunt motors, series motors and compound motors. Both shunt and series types have only one field winding wound on the core of each pole of the motor. The compound type has two separate field windings wound on the core of each pole. The

performance of a d.c. motor can be judged from its characteristic curves known as motor characteristics, following are the three important characteristics of a d.c. motor:

**(i) Torque and Armature current characteristic ( $T_a/I_a$ )**

It is the curve between armature torque  $T_a$  and armature current  $I_a$  of a d.c. motor. It is also known as electrical characteristic of the motor.

**(ii) Speed and armature current characteristic ( $N/I_a$ )**

It is the curve between speed  $N$  and armature current  $I_a$  of a d.c. motor. It is very important characteristic as it is often the deciding factor in the selection of the motor for a particular application.

**(iii) Speed and torque characteristic ( $N/T_a$ )**

It is the curve between speed  $N$  and armature torque  $T_a$  of a d.c. motor. It is also known as mechanical characteristic.

**CHARACTERISTICS OF SHUNT MOTORS**

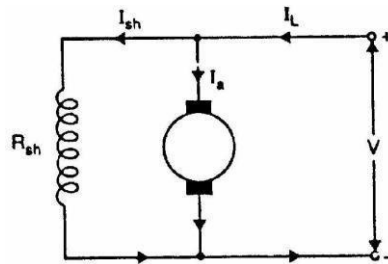


Fig. shows the connections of a d.c. shunt motor. The field current  $I_{sh}$  is constant since the field winding is directly connected to the supply voltage  $V$  which is assumed to be constant. Hence, the flux in a shunt motor is approximately constant.

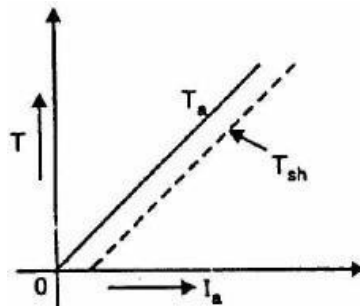
**(i)  $T_a/I_a$  Characteristic.** In a d.c. motor,

$$T_a \propto \phi I_a$$

Since the motor is operating from a constant supply voltage, flux  $\phi$  is constant (neglecting armature reaction).

$$\therefore T_a \propto I_a$$

Hence  $T_a/I_a$  characteristic is a straight line passing through the origin as shown in Fig. The shaft torque ( $T_{sh}$ ) is less than  $T_a$  and is shown by a dotted line. It is clear from the curve that a very large current is required to start a heavy load. Therefore, a shunt motor should not be started on heavy load.

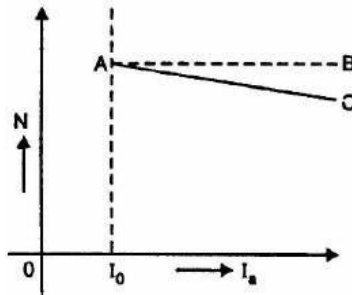


**(ii)  $N/I_a$  Characteristic.** The speed  $N$  of a d.c. motor is given by;

$$N \propto \frac{E_b}{\phi}$$

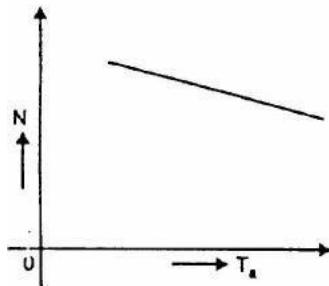
The flux  $\phi$  and back e.m.f.  $E_b$  in a shunt motor are almost constant under normal conditions. Therefore, speed of a shunt motor will remain constant as the armature current varies (dotted line AB

in Fig.). When load is increased,  $E_b (= V - I_a R_a)$  and  $\phi$  decrease due to the armature resistance drop and armature reaction respectively. However,  $E_b$  decreases slightly more than  $\phi$  so that the speed of the motor decreases slightly with load (line AC).



**(iii) N/T<sub>a</sub> Characteristic.**

The curve is obtained by plotting the values of N and T<sub>a</sub> for various armature currents. It may be seen that speed falls somewhat as the load torque increases.



**Conclusions**

Following two important conclusions are drawn from the above characteristics:

- (i) There is slight change in the speed of a shunt motor from no-load to fullload. Hence, it is essentially a constant-speed motor.
- (ii) The starting torque is not high because  $T_a \propto I_a$ .

**CHARACTERISTICS OF SERIES MOTORS**

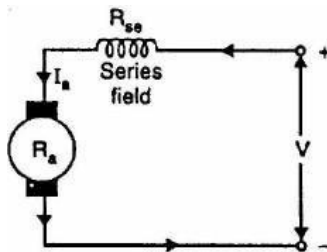


Fig. shows the connections of a series motor. Note that current passing through the field winding is the same as that in the armature. If the mechanical load on the motor increases, the armature current also increases. Hence, the flux in a series motor increases with the increase in armature current and vice-versa.

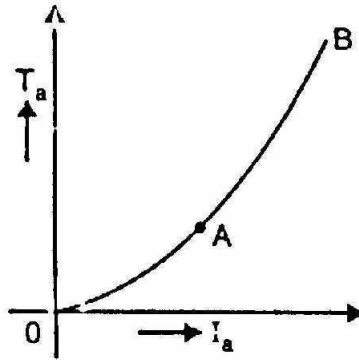
(i) **T<sub>a</sub>/I<sub>a</sub> Characteristic.** We know that:

$$T_a \propto \phi I_a$$

Up to magnetic saturation,  $\phi \propto I_a$  so that  $T_a \propto I_a^2$

After magnetic saturation,  $\phi$  is constant so that  $T_a \propto I_a$

Thus up to magnetic saturation, the armature torque is directly proportional to the square of armature current. If  $I_a$  is doubled,  $T_a$  is almost quadrupled.



Therefore,  $T_a/I_a$  curve upto magnetic saturation is a parabola (portion OA of the curve in Fig.). However, after magnetic saturation, torque is directly proportional to the armature current. Therefore,  $T_a/I_a$  curve after magnetic saturation is a straight line (portion AB of the curve).

It may be seen that in the initial portion of the curve (i.e. upto magnetic saturation),  $T_a \propto I_a^2$ . This means that starting torque of a d.c. series motor will be very high as compared to a shunt motor (where that  $T_a \propto I_a$ ).

**(ii)  $N/I_a$  Characteristic.**

The speed  $N$  of a series motor is given by;

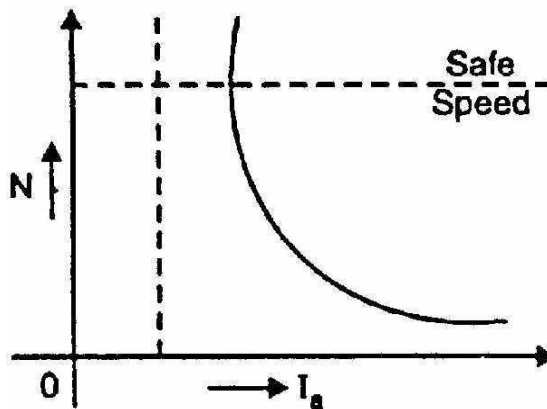
$$N \propto \frac{E_b}{\phi} \quad \text{where} \quad E_b = V - I_a(R_a + R_{se})$$

When the armature current increases, the back e.m.f.  $E_b$  decreases due to  $I_a(R_a + R_{se})$  drop while the flux  $\phi$  increases. However,  $I_a(R_a + R_{se})$  drop is quite small under normal conditions and may be neglected.

$$\therefore N \propto \frac{1}{\phi}$$

$$\propto \frac{1}{I_a} \text{ upto magnetic saturation}$$

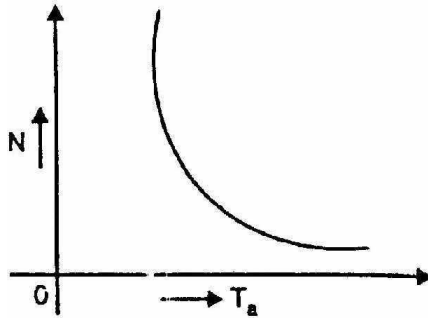
Thus, up to magnetic saturation, the  $N/I_a$  curve follows the hyperbolic path as shown in Fig. After saturation, the flux becomes constant and so does the speed.



**(iii)  $N/T_a$  Characteristic.**

The  $N/T_a$  characteristic of a series motor is shown in Fig. It is clear that series motor develops high torque at low speed and vice-versa. It is because an increase in torque requires an increase in armature current, which is also the field current. The result is that flux is strengthened and hence the speed drops ( $\square N \propto 1/\phi$ ). Reverse happens should the torque be low.





**Conclusions**

- (i) It has a high starting torque because initially  $T_a \propto I_a^2$ .
- (ii) It is a variable speed motor ( $N/I_a$  curve) i.e., it automatically adjusts the speed as the load changes. Thus if the load decreases, its speed is automatically raised and vice-versa.
- (iii) At no-load, the armature current is very small and so is the flux. Hence, the speed rises to an excessive high value ( $\square N \propto 1/\phi$ ). This is dangerous for the machine which may be destroyed due to centrifugal forces set up in the rotating parts. Therefore, a series motor should never be started on no-load. However, to start a series motor, mechanical load is first put and then the motor is started.

**Note.** The minimum load on a d.c. series motor should be great enough to keep the speed within limits. If the speed becomes dangerously high, then motor must be disconnected from the supply.

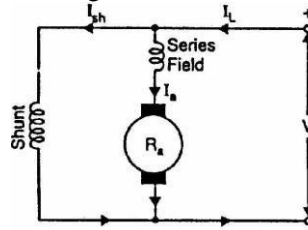
**Compound Motors**

A compound motor has both series field and shunt field. The shunt field is always stronger than the series field. Compound motors are of two types:

- (i) *Cumulative-compound motors* in which series field aids the shunt field.
  - (ii) *Differential-compound motors* in which series field opposes the shunt field.
- Differential compound motors are rarely used due to their poor torque characteristics at heavy loads.

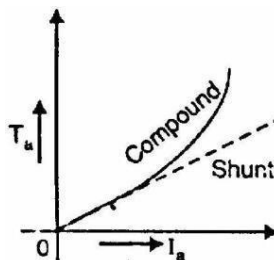
**Characteristics of Cumulative Compound Motors**

Fig. shows the connections of a cumulative-compound motor. Each pole carries a series as well as shunt field winding; the series field aiding the shunt field.



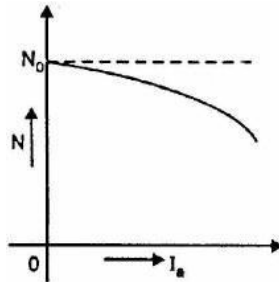
**(i)  $T_a/I_a$  Characteristic.**

As the load increases, the series field increases but shunt field strength remains constant. Consequently, total flux is increased and hence the armature torques ( $\square T_a \propto \phi I_a$ ). It may be noted that torque of a cumulative-compound motor is greater than that of shunt motor for a given armature current due to series field.



**(ii) N/I<sub>a</sub> Characteristic.**

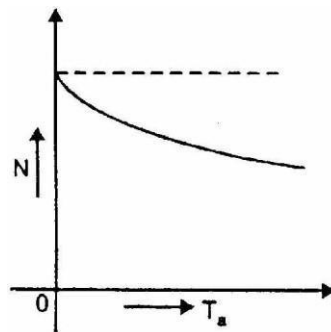
As explained above, as the load increases, the flux per pole also increases. Consequently, the speed ( $N \propto 1/\phi$ ) of the motor falls as the load increases (See Fig.). It may be noted that as the load is added, the increased amount of flux causes the speed to decrease more than does the speed of a shunt motor. Thus the speed regulation of a cumulative compound motor is poorer than that of a shunt motor.



**Note:** Due to shunt field, the motor has a definite no load speed and can be operated safely at no-load.

**(iii) N/T<sub>a</sub> Characteristic.**

Fig. shows N/T<sub>a</sub> characteristic of a cumulative compound motor. For a given armature current, the torque of a cumulative compound motor is more than that of a shunt motor but less than that of a series motor.

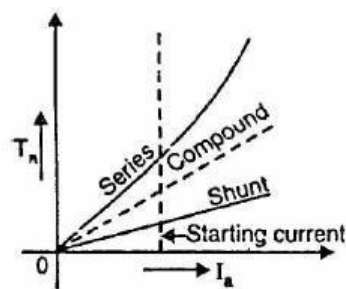
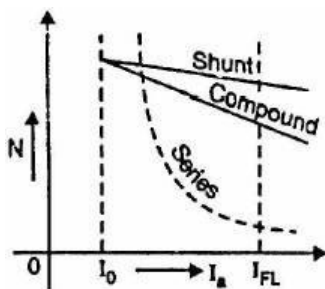


**Conclusions**

A cumulative compound motor has characteristics intermediate between series and shunt motors.

- (i) Due to the presence of shunt field, the motor is prevented from running away at no-load.
- (ii) Due to the presence of series field, the starting torque is increased.

**Comparison of Three Types of Motors**



- (i) The speed regulation of a shunt motor is better than that of a series motor. However, speed regulation of a cumulative compound motor lies between shunt and series motors.
- (ii) For a given armature current, the starting torque of a series motor is more than that of a shunt motor. However, the starting torque of a cumulative compound motor lies between series and shunt motors.
- (iii) Both shunt and cumulative compound motors have definite no-load speed. However, a series motor has dangerously high speed at no-load.

## APPLICATIONS OF D.C. MOTORS

### 1. Shunt motors

The characteristics of a shunt motor reveal that it is an approximately constant speed motor. It is, therefore, used

- (i) where the speed is required to remain almost constant from no-load to full-load
- (ii) where the load has to be driven at a number of speeds and any one of which is required to remain nearly constant

*Industrial use:* Lathes, drills, boring mills, shapers, spinning and weaving machines etc.

### 2. Series motors

It is a variable speed motor i.e., speed is low at high torque and vice-versa. However, at light or no-load, the motor tends to attain dangerously high speed. The motor has a high starting torque. It is, therefore, used

- (i) where large starting torque is required e.g., in elevators and electric traction
- (ii) where the load is subjected to heavy fluctuations and the speed is automatically required to reduce at high torques and vice-versa

*Industrial use:* Electric traction, cranes, elevators, air compressors, vacuum cleaners, hair drier, sewing machines etc.

### 3. Compound motors

Differential-compound motors are rarely used because of their poor torque characteristics. However, cumulative-compound motors are used where a fairly constant speed is required with irregular loads or suddenly applied heavy loads.

*Industrial use:* Presses, shears, reciprocating machines etc.

## Necessity of D.C. Motor Starter

At starting, when the motor is stationary, there is no back e.m.f. in the armature. Consequently, if the motor is directly switched on to the mains, the armature will draw a heavy current ( $I_a = V/R_a$ ) because of small armature resistance.

As an example, 5 H.P., 220 V shunt motor has a full-load current of 20 A and an armature resistance of about 0.5  $\Omega$ . If this motor is directly switched on to supply, it would take an armature current of  $220/0.5 = 440$  A which is 22 times the full-load current. This high starting current may result in:

- (i) burning of armature due to excessive heating effect,
- (ii) damaging the commutator and brushes due to heavy sparking,
- (iii) excessive voltage drop in the line to which the motor is connected. The result is that the operation of other appliances connected to the line may be impaired and in particular cases, they may refuse to work.

In order to avoid excessive current at starting, a variable resistance (known as starting resistance) is inserted in series with the armature circuit. This resistance is gradually reduced as the motor gains speed (and hence  $E_b$  increases) and eventually it is cut out completely when the motor has attained full speed. The value of starting resistance is generally such that starting current is limited to 1.25 to 2 times the full-load current.

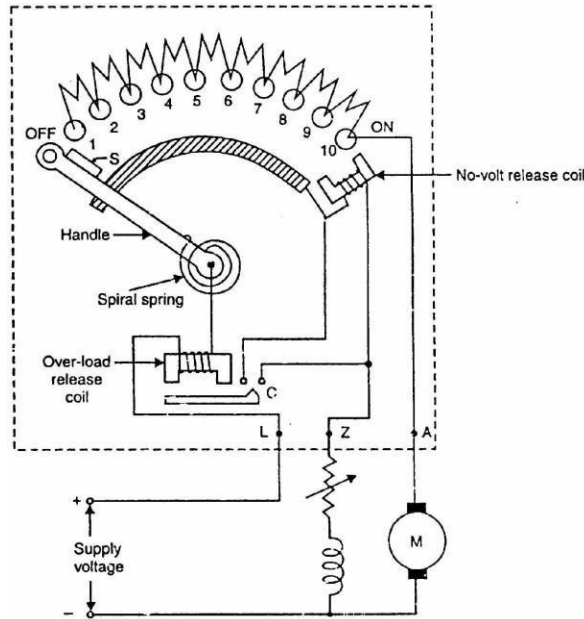
### Types of D.C. Motor Starters

The stalling operation of a d.c. motor consists in the insertion of external resistance into the armature circuit to limit the starting current taken by the motor and the removal of this resistance in steps as the motor accelerates. When the motor attains the normal speed, this resistance is totally cut out of the armature circuit. It is very important and desirable to provide the starter with protective devices to enable the starter arm to return to OFF position

- (i) when the supply fails, thus preventing the armature being directly across the mains when this voltage is restored. For this purpose, we use no-volt release coil.
- (ii) when the motor becomes overloaded or develops a fault causing the motor to take an excessive current. For this purpose, we use overload release coil.

### 1. Three-Point Starter

This type of starter is widely used for starting shunt and compound motors.



- It is so called because it has three terminals L, Z and A.
- The starter consists of starting resistance divided into several sections and connected in series with the armature.
- The tapping points of the starting resistance are brought out to a number of studs.
- The three terminals L, Z and A of the starter are connected respectively to the positive line terminal, shunt field terminal and armature terminal.
- The other terminals of the armature and shunt field windings are connected to the negative terminal of the supply.
- The no-volt release coil is connected in the shunt field circuit.
- One end of the handle is connected to the terminal L through the over-load release coil.
- The other end of the handle moves against a spiral spring and makes contact with each stud during starting operation, cutting out more and more starting resistance as it passes over each stud in clockwise direction.

#### Operation

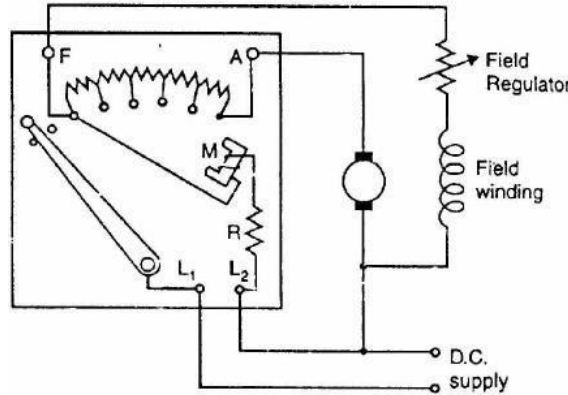
- To start with, the d.c. supply is switched on with handle in the OFF position.
- The handle is now moved clockwise to the first stud. As soon as it comes in contact with the first stud, the shunt field winding is directly connected across the supply, while the whole starting resistance is inserted in series with the armature.
- As the handle is gradually moved over to the final stud, the starting resistance is cut out of the armature circuit in steps. The handle is now held magnetically by the no-volt release coil which is energized by shunt field current.
- If the supply voltage is suddenly interrupted or if the field excitation is accidentally cut, the no-volt release coil is demagnetized and the handle goes back to the OFF position under the pull of the spring. If no-volt release coil were not used, then in case of failure of supply, the handle would remain on the final stud. If then supply is restored, the motor will be directly connected across the supply, resulting in an excessive armature current.
- If the motor is over-loaded (or a fault occurs), it will draw excessive current from the supply. This current will increase the ampere-turns of the over-load release coil and pull the armature C, thus short-circuiting the no-volt release coil. The no-volt coil is demagnetized and the handle is pulled to the OFF position by the spring. Thus, the motor is automatically disconnected from the supply.

**Drawback**

In a three-point starter, the no-volt release coil is connected in series with the shunt field circuit so that it carries the shunt field current. While exercising speed control through field regulator, the field current may be weakened to such an extent that the no-volt release coil may not be able to keep the starter arm in the ON position. This may disconnect the motor from the supply when it is not desired. This drawback is overcome in the four point starter.

**2. Four-Point Start**

In a four-point starter, the no-volt release coil is connected directly across the supply line through a protective resistance R. Fig. shows the schematic diagram of a 4-point starter for a shunt motor (over-load release coil omitted for clarity of the figure).



Now the no-volt release coil circuit is independent of the shunt field circuit. Therefore, proper speed control can be exercised without affecting the operation of no volt release coil.

Only difference between a three-point starter and a four-point starter is the manner in which no-volt release coil is connected. However, the working of the two starters is the same. It may be noted that the three point starter also provides protection against an open field circuit. This protection is not provided by the four-point starter.

**EFFICIENCY OF A D.C. MACHINE**

The power that a d.c. machine receives is called the input and the power it gives out is called the output. Therefore, the efficiency of a d.c. machine, like that of any energy-transferring device, is given by;

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \tag{i}$$

$$\text{Output} = \text{Input} - \text{Losses} \quad \text{and} \quad \text{Input} = \text{Output} + \text{Losses}$$

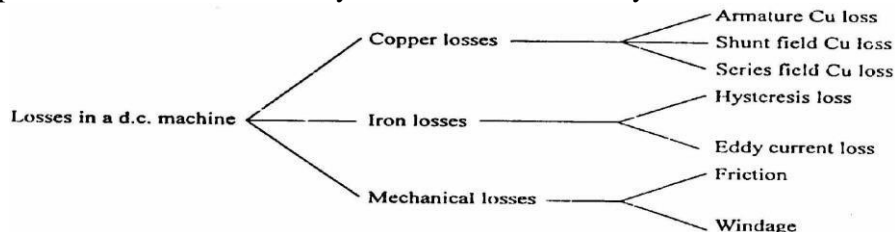
Therefore, the efficiency of a d.c. machine can also be expressed in the following forms:

$$\text{Efficiency} = \frac{\text{Input} - \text{Losses}}{\text{Input}} \tag{ii}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Losses}} \tag{iii}$$

**Losses in a D.C. Machine**

The losses in a d.c. machine (generator or motor) may be divided into three classes viz (i) **copper losses** (ii) **iron or core losses** and (iii) **mechanical losses**. All these losses appear as heat and thus raise the temperature of the machine. They also lower the efficiency of the machine.



### 1. Copper losses

These losses occur due to currents in the various windings of the machine.

- (i) Armature copper loss =  $I_a^2 R_a$
- (ii) Shunt field copper loss =  $I_{sh}^2 R_{sh}$
- (iii) Series field copper loss =  $I_{se}^2 R_{se}$

### 2. Iron or Core losses

These losses occur in the armature of a d.c. machine and are due to the rotation of armature in the magnetic field of the poles. They are of two types viz., (i) hysteresis loss (ii) eddy current loss.

### 3. Mechanical losses

These losses are due to friction and windage.

- (i) friction loss e.g., bearing friction, brush friction etc.
- (ii) windage loss i.e., air friction of rotating armature.

These losses depend upon the speed of the machine. But for a given speed, they are practically constant.

**Note.** Iron losses and mechanical losses together are called stray losses.

### Constant and Variable Losses

The losses in a d.c. generator (or d.c. motor) may be sub-divided into (i) constant losses (ii) variable losses.

#### (i) Constant losses

Those losses in a d.c. generator which remain constant at all loads are known as constant losses. The constant losses in a d.c. generator are:

- (a) iron losses
- (b) mechanical losses
- (c) shunt field losses

#### (ii) Variable losses

Those losses in a d.c. generator which vary with load are called variable losses.

The variable losses in a d.c. generator are:

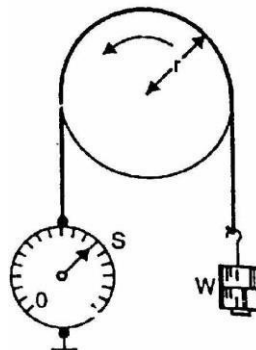
- (a) Copper loss in armature winding
- (b) Copper loss in series field winding

### Total losses = Constant losses + Variable losses

**Note.** Field Cu loss is constant for shunt and compound generators.

### Efficiency by Direct Loading (Load test)

In this method, the d.c. machine is loaded and output and input are measured to find the efficiency.



In this method, a brake is applied to a water-cooled pulley mounted on the motor shaft as shown in Fig. One end of the rope is fixed to the floor via a spring balance S and a known mass is suspended at the other end. If the spring balance reading is S kg-Wt and the suspended mass has a weight of W kg-Wt, then,

Net pull on the rope =  $(W - S)$  kg-Wt =  $(W - S) \times 9.81$  newtons

If  $r$  is the radius of the pulley in metres, then the shaft torque  $T_{sh}$  developed by the motor is

$$T_{sh} = (W - S) \times 9.81 \times r \text{ N - m}$$

If the speed of the pulley is  $N$  r.p.m., then,

$$\text{Output power} = \frac{2\pi N T_{sh}}{60} = \frac{2\pi N \times (W - S) \times 9.81 \times r}{60} \text{ watts}$$

Let  $V$  = Supply voltage in volts

$I$  = Current taken by the motor in amperes

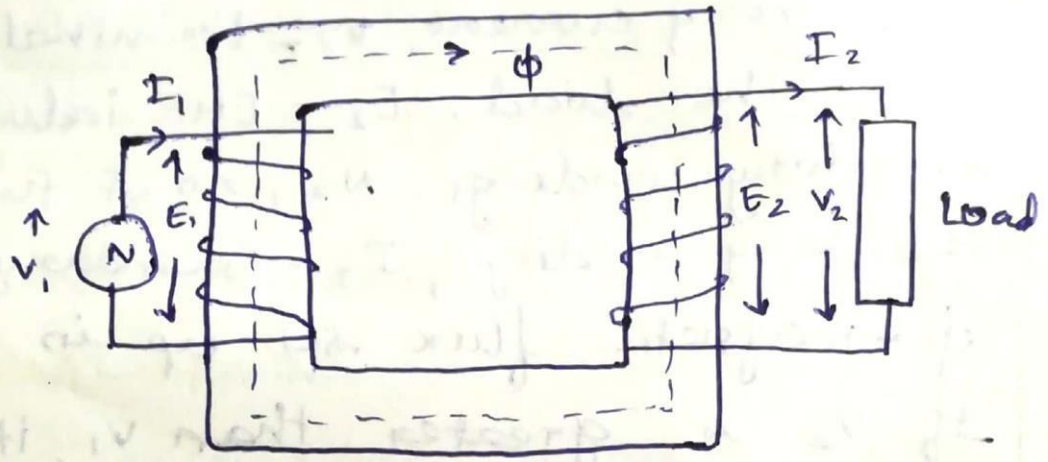
$$\therefore \text{Input to motor} = V I \text{ watts}$$

$$\therefore \text{Efficiency} = \frac{2\pi N(W - S) \times r \times 9.81}{60 \times VI}$$



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MODULE - III  
TRANSFORMERS



Principle of operation of Transformer is Faraday's law of Electro magnetic induction. The transformer is a static device, used either for raising the voltage or lowering the voltage of an AC supply with a corresponding decrease or increase in the current. The transformer consist of two windings; primary and secondary - wound on common laminated magnetic pole. The primary winding connected to ac source and secondary winding connected to load. In general the winding connected to source is called primary winding. The winding connected to load is called secondary winding. Depending upon the no. of turns of primary and secondary. An alternating



emf  $E_2$  is induced in secondary, which causes the secondary current  $I_2$ . Let  $V_1$  be the source voltage,  $E_1$  - EMF induced in primary,  $N_1$  - no. of turns of primary,  $I_1$  - primary current,  $V_2$  - terminal voltage across the load.  $E_2$  - EMF induced in the secondary windings.  $N_2$ , no. of turns of secondary winding,  $I_2$  - secondary current.

$\phi$  - magnetic flux set up in the core

If  $V_2$  is greater than  $V_1$ , it is step up transformer. If  $V_2$  is less than  $V_1$ , it is step down transformer.

An alternating voltage  $V_1$  is applied to the primary and alternating flux  $\phi$  is set up in the core. This flux links with both the windings and induces EMF  $E_1$  and  $E_2$ .

$$E_1 = -N_1 \frac{d\phi}{dt}$$

$$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\text{Then } \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

If  $N_2 > N_1$ , then  $E_2 > E_1$ ,  $V_2 > V_1$  and  $I_2 < I_1$  - when the transformer is a step up transformer.

If  $N_2 < N_1$ ,  $E_2 < E_1$ ,  $V_2 < V_1$  and  $I_2 > I_1$  when the transformer is a step down transformer.



When the secondary EMF  $E_2$  is induced, it will cause current  $I_2$  to flow through the load, thus the transformer is enabled to transfer AC power from one circuit to another with change in voltage level.

### Features of transformer:

1. There is no connection b/w primary and secondary
2. AC power is transferred from primary to secondary through magnetic flux.
3. There is no change in frequency i.e. output power has same frequency as input power.

### Losses in a transformer:

1. Copper loss - (Windings of transformer)
2. Core loss - (Mix of Eddy current and hysteresis)

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### EMF eqn of transformer:

$$\text{Flux } \phi = \phi_m \sin \omega t$$

The sinusoidal flux  $\phi$  is produced when an alternating voltage  $V_1$  of frequency  $f$  is supplied to the primary winding

$$E_1 = -N_1 \frac{d\phi}{dt}$$



$$e_1 = -N_1 \frac{d}{dt} (\phi_m \sin \omega t)$$

$$e_1 = -N_1 \omega \phi_m \cos \omega t$$

$$e_1 = -N_1 \omega \phi_m \sin(90 - \omega t)$$

$$e_1 = N_1 \omega \phi_m \sin(\omega t - 90) \quad (\omega = 2\pi f)$$

Maximum emf induced in Primary winding  $E_{m1} = N_1 \omega \phi_m$

$$E_{m1} = N_1 2\pi f \phi_m$$

Root mean sq. value (rms) of primary e

$$E_1 = \frac{E_{m1}}{\sqrt{2}}$$

$$E_{rms} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}} = \sqrt{2} \pi f N_1 \phi_m$$

$$E_1 = 4.44 f N_1 \phi_m$$

$$E_2 = 4.44 f N_2 \phi_m$$

Transformation ratio

$$k = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{I_1}{I_2}$$

Q. A 2000/200 V 20 kVA transformer has 66 turns in the secondary. Calculate the primary turns, primary and 2<sup>o</sup> full load current. Neglect the losses.

A. Given:

$$V_1 = 2000 \text{ V} \quad V_2 = 200 \text{ V} \quad N_2 = 66.$$

Power = 20 kVA

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$\therefore N_1 = \frac{N_2 \times V_1}{V_2}$$
$$= \frac{66 \times 2000}{200}$$

$$N_1 = \underline{\underline{660 \text{ turns}}}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = 0.1$$

$$I_1 = 0.1 I_2$$

Power is constant at input power and output

$$P_2 = 20 \text{ kVA.}$$

$$I_2 = \frac{20 \text{ kVA}}{V_2} = \frac{20 \text{ kVA}}{200} = \underline{\underline{100 \text{ A}}}$$

$$\therefore I_1 = 0.1 I_2$$

$$I_1 = \underline{\underline{10 \text{ A}}}$$



Q. An ideal transformer 25 kVA has 500 turns on the primary windings and 40 turns on secondary winding. Primary is connected to 3000V, 50 Hz supply. Calculate primary and secondary currents on full load, secondary emf and max core flux.

A.  $P = 25 \text{ kVA}$ .

$N_1 = 500 \text{ turns}$

$N_2 = 40 \text{ turns}$

$V_1 = 3000 \text{ V}$

$\frac{N_2}{N_1} = \frac{V_2}{V_1}$

$\frac{N_2}{N_1} = \frac{40}{500} = 0.08$

$V_2 = V_1 \times 0.08 = 240$

$\frac{I_1}{I_2} = 0.08$

$I_1 = \frac{P}{V_1} = \frac{25000}{3000}$

$= 8.33 \text{ A}$

$\therefore I_2 = \frac{8.33}{0.08} = 104.12 \text{ A}$

$E_1 = 4.44 f N_1 \phi_m$

As there are no losses,

$V_1 = E_1 = 3000 \text{ V}$

$\frac{N_2}{N_1} = \frac{E_2}{E_1}$

$\therefore E_2 = 0.08 \times 3000 = 240 \text{ V}$

$\therefore \phi_m = \frac{E_1}{4.44 \times 50 \times 500} = \frac{3000}{111000}$

$\phi_m = 0.027 \text{ wb}$



A. A single phase 50 Hz transformer has square core of 20 cm side. The permissible maximum flux density in the core is 1 wb/m<sup>2</sup>. Calculate the no. of turns on the high voltage side and low voltage side for a 3000/220 v ratio. Assume net iron length to be 0.9 x Gross iron length

A. Given:

$f = 50 \text{ Hz}$        $\phi/A = 1 \text{ wb/m}^2 = B$

$V_1 = 3000 \text{ V}$        $V_2 = 220 \text{ V}$

To find:

$N_1$  and  $N_2$ .

$\phi = A \times B = 1 \text{ wb/m}^2$

~~$\phi = 1 \text{ wb/m}^2$~~

$E_1 = V_1 = 3000 \text{ V}$

~~$\phi = 0.84 \text{ wb}$~~

$E_1 = 4.44 f N_1 \phi_m$

$A = (\text{Iron length}) \times 0.2$

$\therefore N_1 = \frac{3000}{4.44 \times 50 \times 0.036}$

$= (0.9 \times 0.2) \times 0.2$

$\phi = 0.036 \text{ wb}$

$N_1 = \underline{\underline{378 \text{ turns}}}$

$\therefore N_2 = \frac{E_2 \times N_1}{E_1} = \frac{220 \times 378}{3000}$

~~$= 27.72$~~

$= \underline{\underline{28 \text{ turns}}}$



sinusoidal flux  $0.02 \text{ wb}$  links with 55 turns of transformer secondary. Calculate the rms value of induced emf in the secondary.  $f = 50 \text{ Hz}$

A. Given:

$$\phi_{\max} = 0.02 \quad N_2 = 55$$

$$E_{\text{rms}} = 4.44 f N_2 \phi_{\max}$$

$$= 4.44 \times 50 \times 55 \times 0.02$$

$$= 244.2 \text{ V}$$

$$\text{rms value} = \frac{E_z}{\sqrt{2}} = \underline{\underline{172.675 \text{ V}}}$$

Losses in a transformer:

1) Core loss or iron loss:

These consist of hysteresis and eddy current losses and occur in transformer core due to the alternating flux.

$$\text{Hysteresis loss} = K_h f B_m^{1.6} \text{ watts/m}^3$$

$$\text{Eddy Current loss} = K_e f^2 B_m^2 t^2 \text{ watts}$$

Since frequency is constant voltage supply is constant.

$\therefore f$  and  $B_m$  are constant.

constant is determined by Open circuit

Hence iron and core losses practically same at all loads. Core or iron loss is = Hysteresis loss + eddy current loss = constant



It can be reduced by using steel of high silicon content. Eddy current loss can be reduced by using core of thin lamination.

## 2) Copper loss:

These occur in both primary and secondary windings due to their ohmic resistances. This can be determined by short circuit test.

$$\text{Copper loss} = I^2 R$$

$$= I_1^2 R_1 + I_2^2 R_2$$

∴ Total losses is equal to ~~total~~ <sup>constant</sup> loss + copper loss.

## Efficiency of transformer

$$\text{Efficiency } \eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

1. In a 50kVA transformer, iron loss is 500w and full load copper loss is 800w. Find the efficiency at full load.

A.  $P = 50 \text{ kVA}$ .

$$\text{Iron loss} = 500 \text{ w}$$

$$\text{Copper loss} = 800 \text{ w}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{50000}{50000 + 800 + 500} = 97.46\%$$



2. In above question, the power factor is, Find efficiency

A. Efficiency = 
$$\frac{\text{Output power} \times \text{powerfactor}}{(\text{Output power} \times \text{powerfactor}) + 500 + 800}$$

$$= \frac{50000 \times 0.8}{(50000 \times 0.8) + 1300}$$

$$= \underline{\underline{96.85\%}}$$

3. In the above question, the condition is half

A. 
$$\therefore \text{Efficiency} = \frac{\frac{50000}{2} \times 0.8}{\frac{50000}{2} \times 0.8 + \frac{500}{2} + \frac{800}{(2)^2}}$$

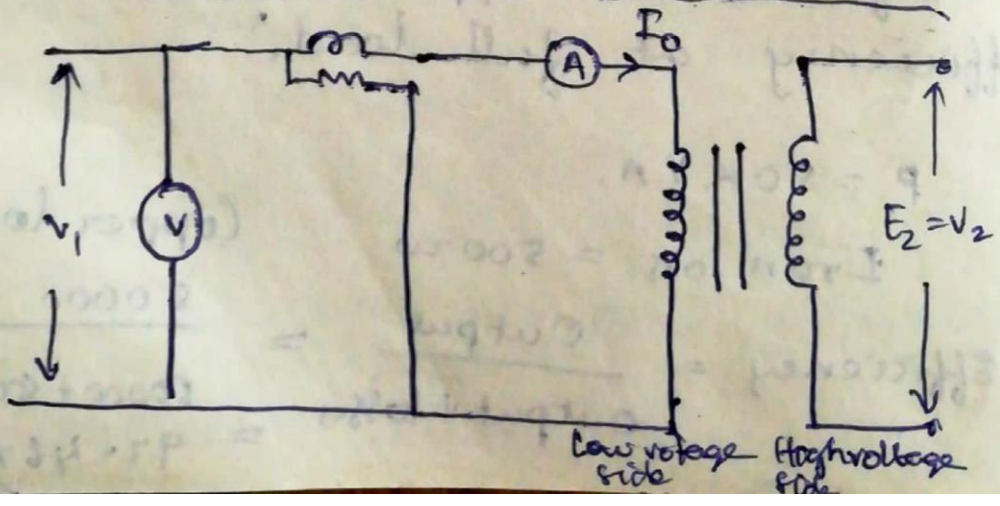
Since

$$= \underline{\underline{96.69\%}}$$

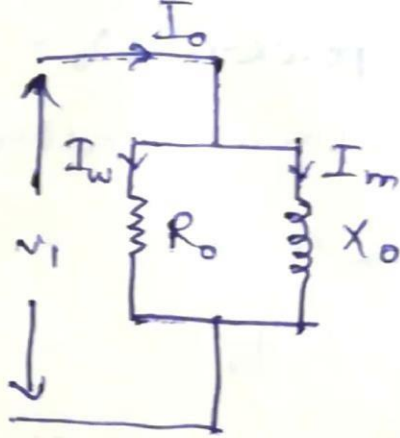
Full load efficiency at any power factor is = 
$$\frac{\text{Full load VA} \times \text{P. factor}}{(\text{Full load VA} \times \text{P. f}) + \text{iron loss} + \text{copper}}$$

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Open circuit Test or No-load test







Open circuit test is conducted to determine iron losses,  $R_0$  value and  $X_0$  values of the transformer. Rated voltage is applied to the primary while secondary is left open circuited. The voltmeter  $V$  connected across the supply will measure the applied voltage  $V_1$ . The ammeter  $A$  connected in series with the transformer measures the no load current  $I_0$ , the wattmeter connected measures the no load power  $w_0$ . By applying the rated voltage to the primary, normal iron losses will occur in the transformer core. Hence the wattmeter will record iron losses and small amount of copper losses which occurs in primary coil. This copper loss is negligible. Hence the wattmeter gives iron losses in the transformer. Iron losses  $P_i$  is equal to wattmeter reading  $w_0$ .  
 No load current = Ammeter reading =  $I_0$   
 Applied voltage = voltmeter reading =  $V_1$



Input power  $W_0 = V_1 I_0 \cos \phi_0$   
 No load power factor,  $\cos \phi_0 = \frac{W_0}{V_1 I_0}$

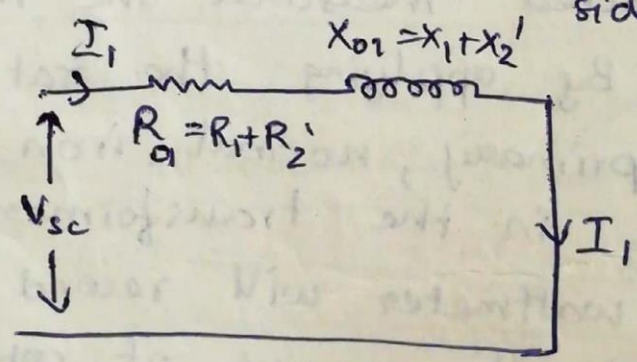
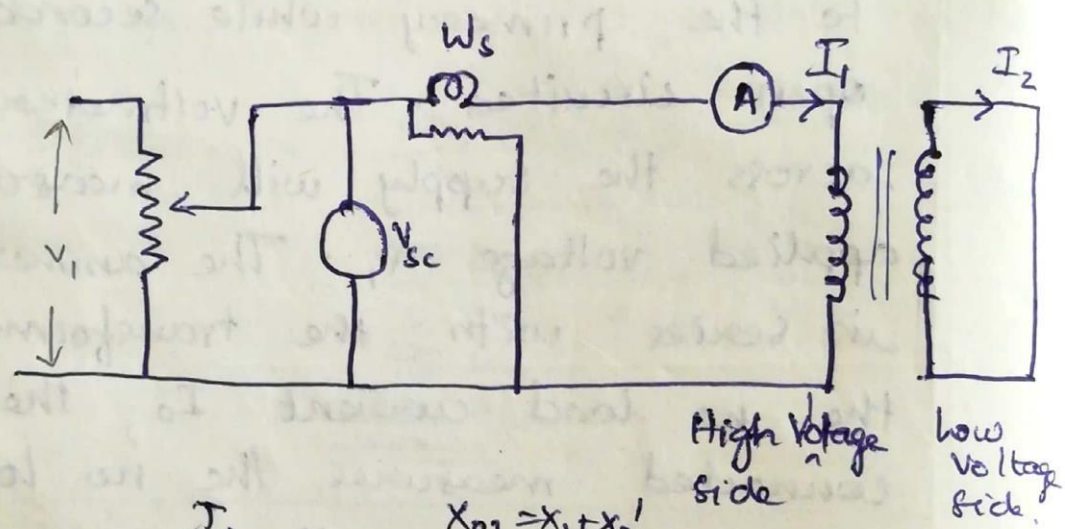
$I_w = I_0 \cos \phi_0$

$I_m = I_0 \sin \phi_0$

$R_0 = \frac{V_1}{I_w}$

$X_0 = \frac{V_1}{I_m}$

Short circuit test or Impedance test



This is conducted to determine  $R_{01}$  or  $R_{02}$ ,  $X_{01}$  or  $X_{02}$  and full load copper losses of the transformer. Secondary is short circuited by a thick conductor and variable voltage is applied to the primary. The input voltage is gradually raised till  $V_{sc}$ , such that the full load current  $I_1$



flows in the primary. As it is short circuited, the input power given is ~~lost~~ and the loss is almost copper loss. Here the iron loss is negligibly small as the  $V_{sc}$  under the short circuit condition is about  $1/10^{\text{th}}$  of normal voltage. Hence wattmeter registers full load copper loss in the transformer winding. The full load copper loss  $P_c = \text{wattmeter reading} = W_s$   
 Applied voltage is  $= \text{voltage reading} = V_{sc}$   
 Full load primary current  $= \text{ammeter reading} = I_1$ .

$$\therefore P_c = I_1^2 R_1 + I_2^2 R_2 =$$

$$P_c = I_1^2 R_1 + I_2'^2 R_2'$$

$$P_c = I_1^2 R_{01} \quad (\text{Referred to primary})$$

$$R_{01} = \frac{P_c}{I_1^2}$$

$R_{01}$  is the total resistance of the transformer referred to primary. The total impedance referred to primary

$$Z_{01} = \frac{V_{sc}}{I_1}$$

The total reactance referred to primary

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$



## All day efficiency

$$\text{All day efficiency} = \frac{\text{kWh output in 24 hrs}}{\text{kWh input in 24 hrs}}$$

Q. A 5 kVA distribution transformer has a full load efficiency of 95% at which copper loss = Iron loss. The transformer is loaded for 24 hrs as given below. No load for 10 hrs,  $\frac{1}{4}$ th full load for 7 hours, half full load for 5 hours and full load for 2 hours. Calculate the all day efficiency of the transformer.

A. Given: Power = 5 kVA.  $\times 1 \text{ A} = 5 \text{ kW}$   
Full load output = 95%.

$$\begin{aligned} \text{Full load input} &= \frac{\text{Output}}{\text{efficiency}} \\ &= \frac{5000}{0.95} \end{aligned}$$

$$\begin{aligned} \text{Full load input} &= 5263.15 \text{ W} \\ &= \underline{\underline{5.263 \text{ kW}}} \end{aligned}$$

$$\begin{aligned} \text{Total loss is} & \text{ input} - \text{output.} \\ &= \underline{\underline{263.15 \text{ W}}} \end{aligned}$$

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j, g g,,,}

t» he-c

0.25 kVA

0.5 kVA

) ku4

Losses at 0.25 kVA

$$131.575 \times \left(\frac{0.25}{5}\right)^2 = 65.78 \text{ W}$$

" at 0.5 kVA.

$$= 131.575 \times \left(\frac{0.5}{5}\right)^2 = 131.575 \text{ W}$$

,> at t kv >

$$t5 \setminus 7^* / \wedge$$

\*\*

To

iron loss for 24 hours = 131.575 x 24

$$3157.8 \text{ Whr} + (0.25 \times 5) + (5 \times 2)$$

$$1.75 + 2.5 + 2$$

6250 Whr 31250 Whr

$$\text{Input} = 6250 + 3157.8 + 164.409 = 9572.209 \text{ Whr}$$

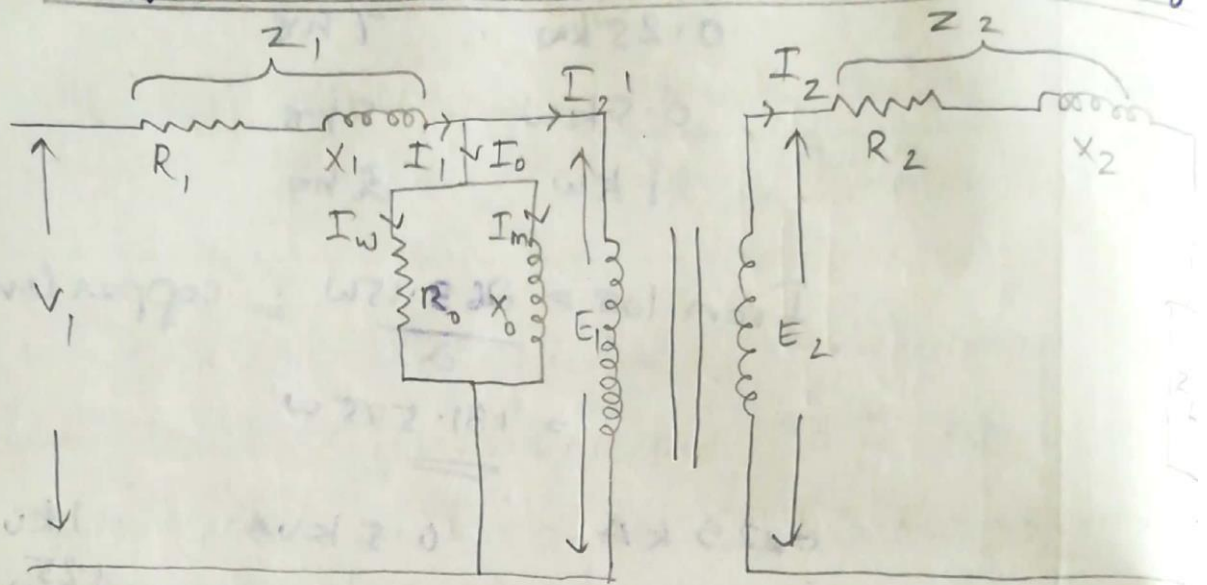
$$6250 + 3157.8 + 822.6 = 10230.4 \text{ Whr}$$

$$\text{Watt-day} = \left(\frac{6250}{9572.209}\right) \times 100 = 65.29\%$$



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# Equivalent Circuit of Loaded Transformer



$R_1$  is primary winding resistance  
 $R_2$  is secondary winding resistance  
 $X_2$  is secondary winding reactance  
 $R_0$  and  $X_0$  is a no load equivalent circuit of transformer  
 Working component of no load current  $I_w$  flows through  $R_0$  and magnetizing component of no load current  $I_m$  flows through  $X_0$ .  $X_0$  is a loss free coil.  
 $E_1$  is the induced EMF.

$$E_1 = V_1 - I_1 Z_1$$

$$X_0 = \frac{E_1}{I_0}$$

$$R_0 = \frac{E_1}{I_w}$$

Primary equivalent of secondary induced EMF  $E_2 = \frac{E_2}{K} = E_1$

Primary equivalent of secondary voltage

$$V_2' = \frac{V_2}{k}$$

Primary equivalent of secondary current

$$I_2' = \frac{I_2}{k}$$

Primary equivalent of secondary resistance

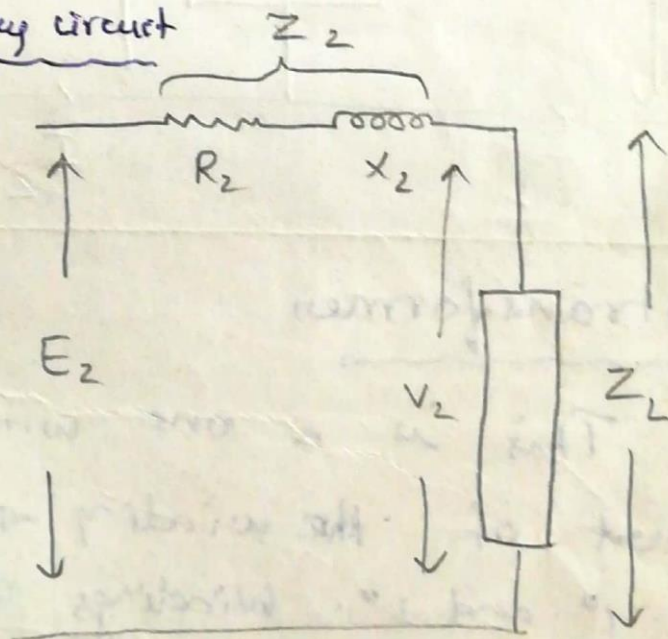
$$R_2' = \frac{R_2}{k^2}$$

Similarly primary <sup>equivalent of  $Z_2$</sup>  leakage reactance

$$X_2' = \frac{X_2}{k^2}$$

Similarly Impedance  $Z_2' = \frac{Z_2}{k^2}$

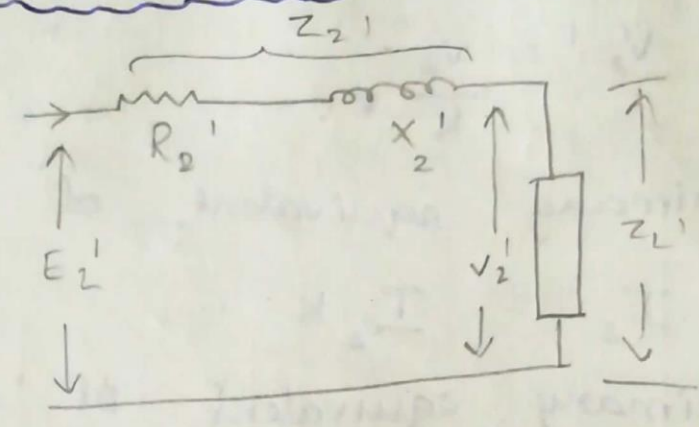
Secondary circuit



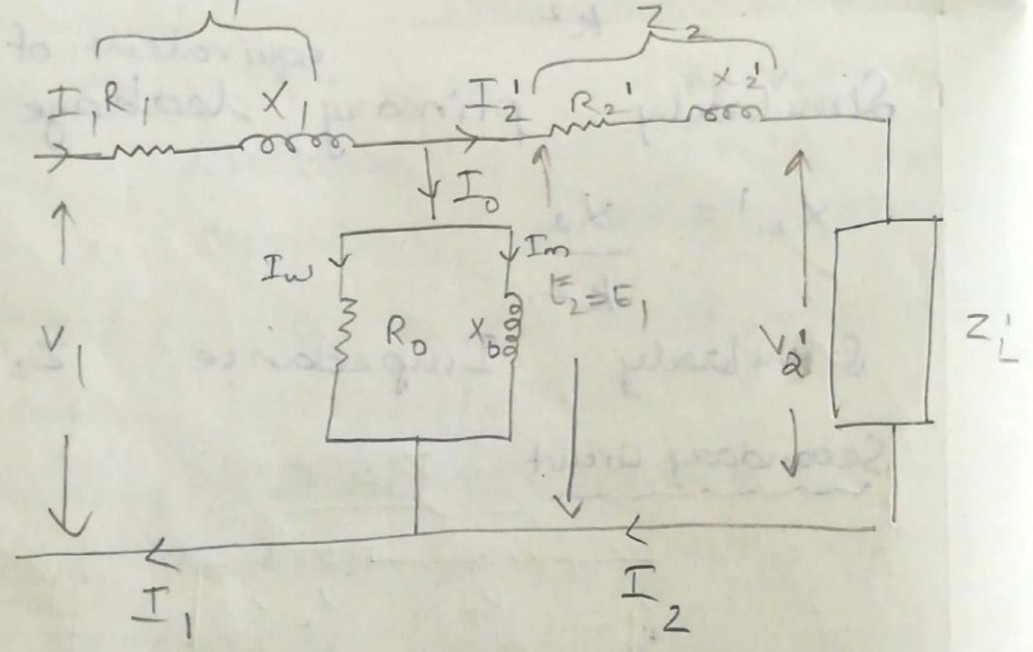
Primary



# Primary Equivalent of Secondary Circuit

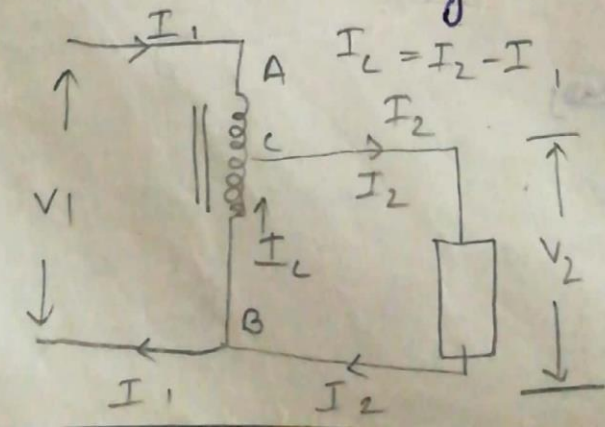


# Simplified form of equivalence circuit

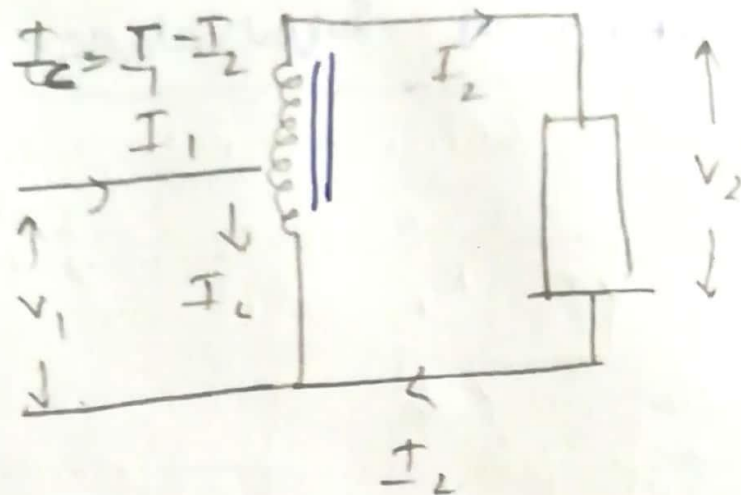


# Autotransformers

This is a one winding transformer. A part of the winding is common to both 1° and 2°. Windings are not electrically isolated.



Step down trans

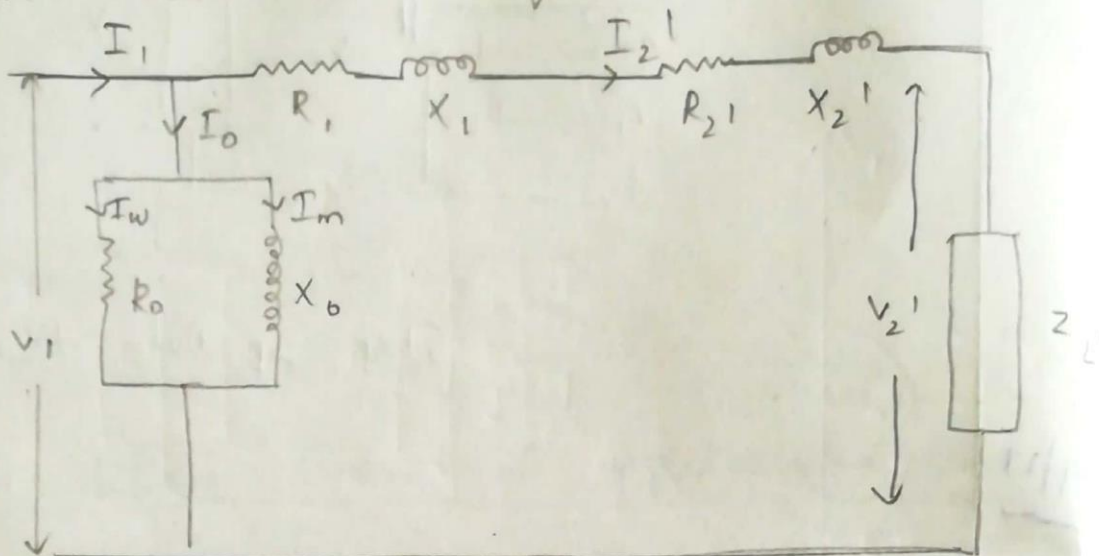


Step up Auto transformer



## Continuation of Equivalence circuit

Equivalence Circuit - Referred to Primary



$$R_1 + R_2' = R_{01}$$

$$X_1 + X_2' = X_{01}$$

when referred to Secondary

$$R_1' = R_1 k^2$$

$$X_1' = X_1 k^2$$

$$V_1' = k V_1$$

$$I_1' = \frac{I_1}{k}$$

$$R_{02} = R_2 + R_1'$$

$$X_{02} = X_2 + X_1'$$

Q. A 4500 / 16000 V, 1500 KVA, 50 Hz transformer has the following parameters.

$$R_1 = 0.03 \Omega$$

$$X_1 = 0.092 \Omega$$

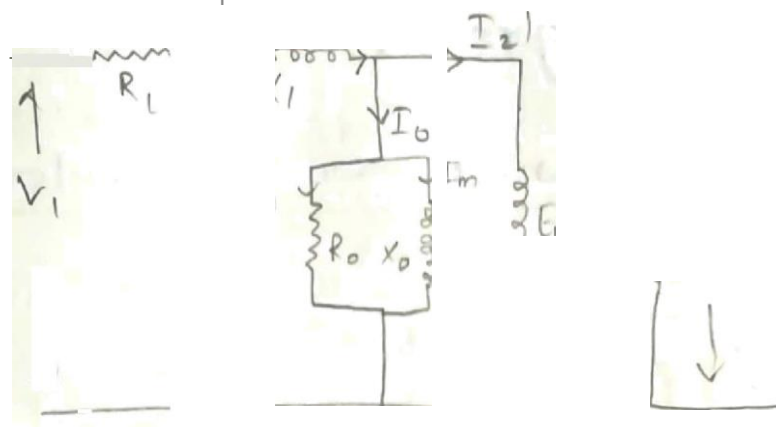
$$R_2 = 0.44 \Omega$$

$$X_2 = 1.34 \Omega$$

$$R_0 = 1688 \Omega$$

$$X_0 = 256 \Omega$$

The transformer is supplying full load at a power factor 0.8 lagging using exact equivalent circuit, find the input current.



en

$V_1 = 4500 \text{ v}$        $V_2 = 16000 \text{ v}$

$\cos \phi = 0.8$        $f = 50 \text{ Hz}$

$\cos \phi = 0.8$

$I_2 = \frac{V_2}{Z}$        $\cos \phi = 0.8$

$V_2$

$I_m = \frac{V_1}{X_0} = \frac{4500}{256} = 17.57 \text{ A}$

$I_w = \frac{V_1}{R_0} = \frac{4500}{1688}$

$E_2 = V_2 + I_2(R_2 + jX_2)$        $353.2 \text{ A}$

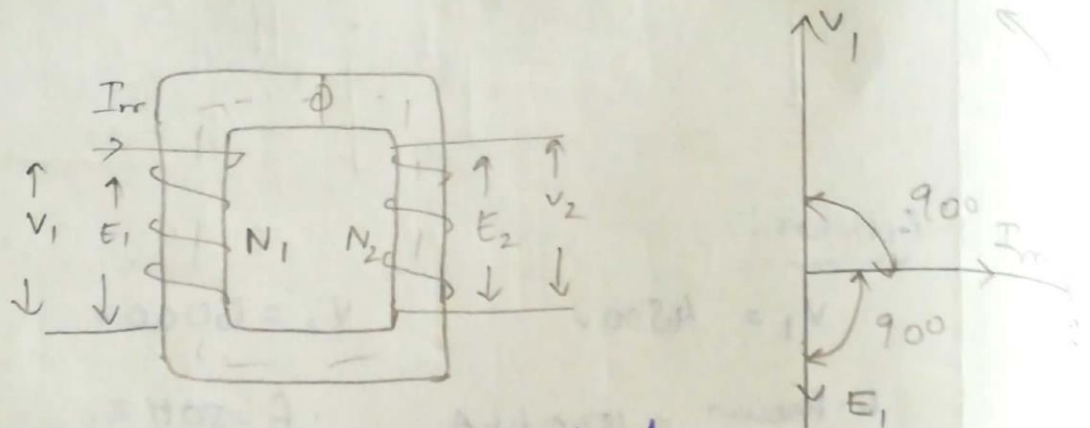
all power



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# Vector Diagram!

## ① Ideal transformer on no-load



Phasor diagram of ideal transformer at no load condition is shown in the figure.

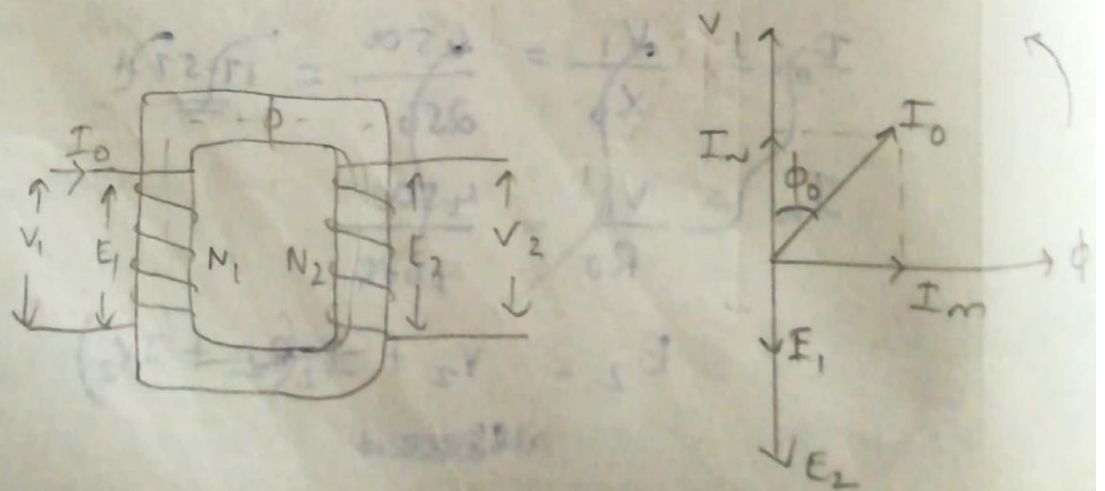
$\phi$  is taken as reference as it is common both the windings. From the equations

$$e_1 = 2\pi f N_1 \phi_m \sin(\omega t - 90^\circ)$$

$$e_2 = 2\pi f N_2 \phi_m \sin(\omega t - 90^\circ)$$

We can see  $e_1$  and  $e_2$  lags behind  $\phi$  by angle  $90^\circ$ .

## ② Practical transformer on no-load.





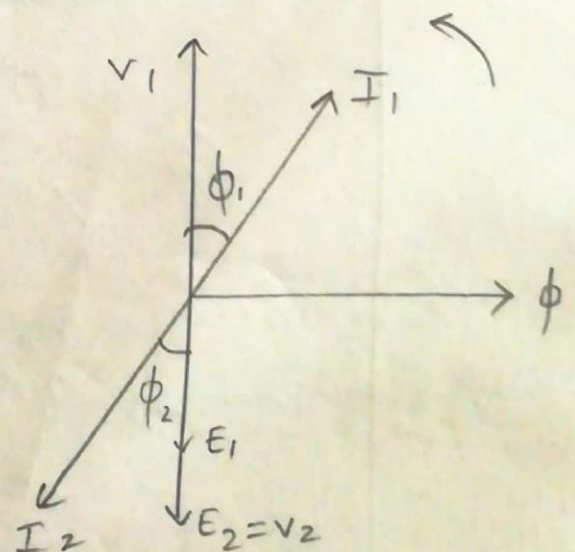
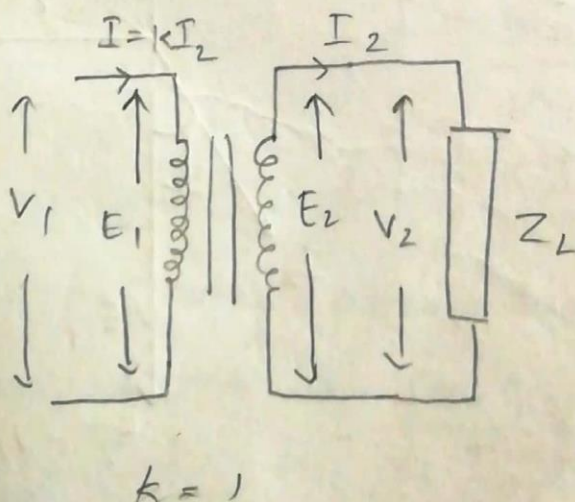
Secondary is open circuited. The primary will draw a small current  $I_0$  to supply iron losses and small amount of copper loss. Hence the primary current  $I_0$  lags behind voltage  $V_1$  by an angle  $\phi_0$ ,  $\phi_0$  is less than  $90^\circ$ . From the phasor diagram,  $I_0$  can be resolved into two rectangular components  $I_w$  and  $I_m$ .

$$I_w = I_0 \cos \phi_0 \quad I_m = I_0 \sin \phi_0$$

$I_w$  is called working component or iron loss component  $I_m$  is called magnetizing component which produces mutual flux  $\phi$  in the core. From the phasor diagram,  $I_0$  = phasor sum of  $I_m$  and  $I_w$   $\therefore I_0$

$$I_0 = \sqrt{I_m^2 + I_w^2} \quad \text{The no load power factor } \cos \phi_0 = \frac{I_w}{I_0}$$

### ③ Ideal transformer on load:





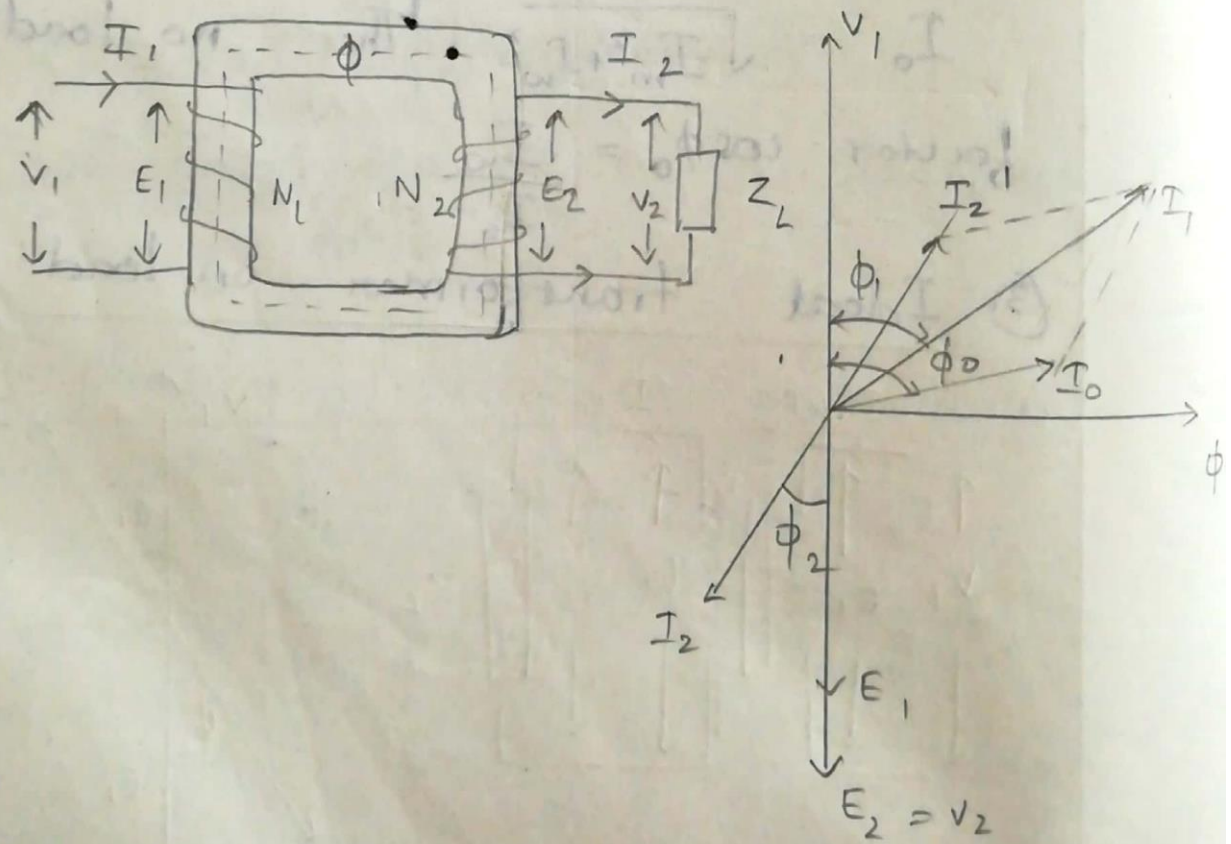
$I_2$  lags behind  $V_2$  by an angle  $\phi_2$ .  $I_1$  given by  $I_1 = KI_2$ , which is anti-phase  $I_1$  lags behind  $V_1$  by angle  $\phi_1$ . Both  $\phi_1$  and  $\phi_2$  are same.  $\therefore \phi_1 = \phi_2$  i.e.  $\cos \phi_1 = \cos \phi_2$ .  $\therefore$  Power factor on 1<sup>o</sup> side is equal to power factor on 2<sup>o</sup> side.

As there is no losses in a ideal transformer input primary power is equal to 2<sup>o</sup> power.

$$V_1 I_1 \cos \phi_1 = V_2 I_2 \cos \phi_2$$

(4) Practical transformer on load

a) No winding resistance & leakage flux





$I_1$  and  $I_2$  lags behind  $V_1$  and  $V_2$   
 $I_2'$  represents primary current to neutralize demagnetizing effect of secondary current  $I_2$ .  $\therefore I_2' = k I_2$

$I_2'$  is anti-phase with  $I_2$ . with the assumption that resistance and leakage reactance of the windings are negligible.  $V_2 = E_2$  and  $V_1 = E_1$ ,  $V_1$  and  $E_1$  are  $180^\circ$  phase shift. Consider an inductive load.  $I_2$  lags  $V_2$  by  $\phi_2$  angle.  $I_1$  must meet two requirements.

1. It should supply no load current  $I_0$  to meet the iron losses in the transformer and to provide flux in the core.
2. It must supply  $I_2'$  current to counteract the demagnetizing effect of secondary current  $I_2$ .  $\therefore N_1 I_2' = N_2 I_2$

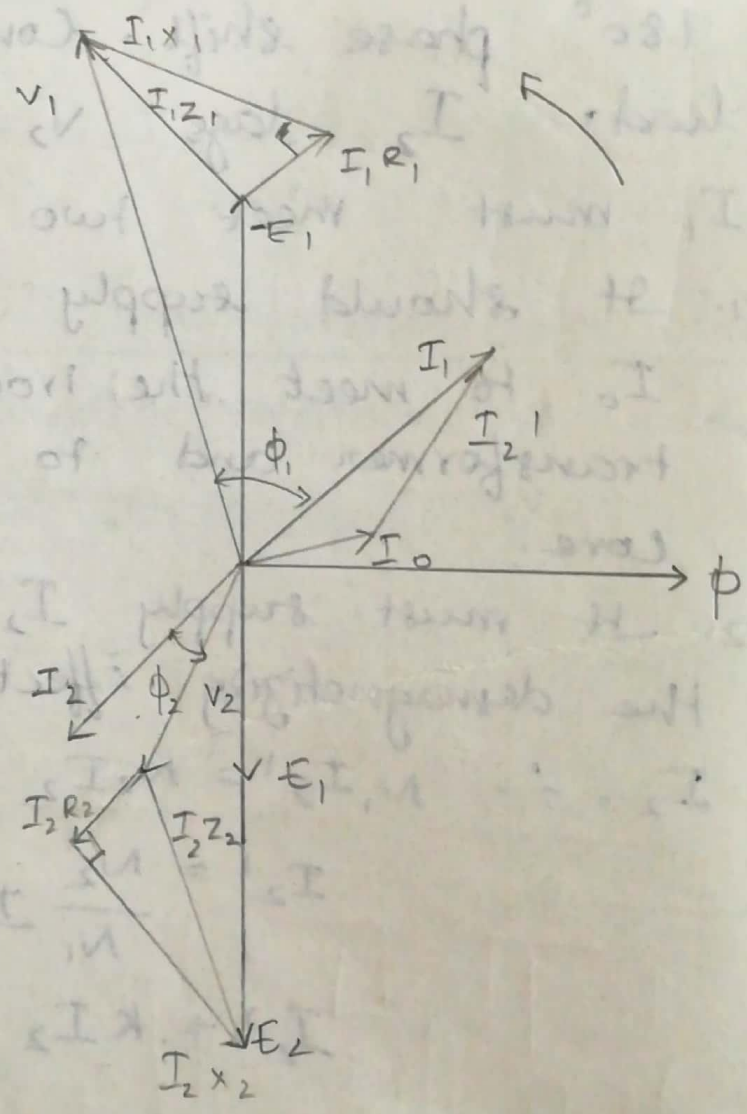
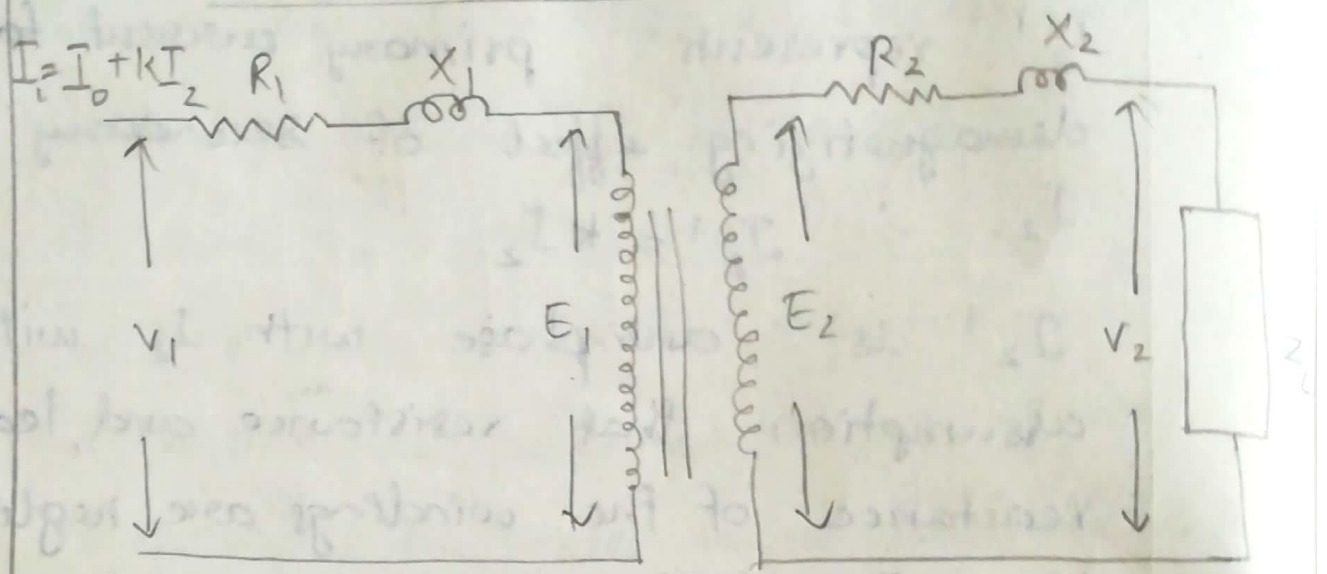
$$I_2' = \frac{N_2}{N_1} I_2$$

$$I_2' = k I_2$$

The primary power factor is  $\cos \phi_1$ .  $2^\circ$  power factor is  $\cos \phi_2$ . Primary input power is equal to  $V_1 I_1 \cos \phi_1$   
 $2^\circ$  output power  $P_2 = V_2 I_2 \cos \phi$



h)(b) With resistance and Leakage resistance



Voltage drop across  $r_1$  and  $x_1$  is considered here so that  $E_1$  is less than  $V_1$ . Similarly voltage drop occurs in  $R_2$  and  $X_2$ .  $\therefore V_2$  is less than  $E_2$ . Considering inductive load.  $I_2$  lags behind  $V_2$ .

The total primary current  $I_1$  must meet two requirements

1) Must supply no load current  $I_0$ , to meet iron losses in the transformer and to provide flux in the core.

2) Must supply  $I_2'$  to counteract demagnetizing effect of secondary current  $I_2$ . The counter EMF opposing  $V_1$  is  $-E_1$

$$N_1 I_2' = N_2 I_2$$

$$\therefore I_2' = \frac{N_2}{N_1} I_2 = k I_2$$

$$\therefore I_1 = I_2' + I_0 \quad \text{where } I_2' = k I_2$$

$$\text{Load power factor} = \cos \phi_2$$

$$\text{Primary " " } = \cos \phi_1$$

$$\text{Input power to the transformer } P_1 = V_1 I_1 \cos \phi_1$$

$$\text{Output power } P_2 = V_2 I_2 \cos \phi_2$$

$$\text{From phasor diagram, } I_1 z_1 = I_1 R_1 + I_1 j X_1$$

$$V_1 = -E_1 + I_1 R_1 + I_1 j X_1$$

$$V_1 = -E_1 + I_1 (R_1 + j X_1)$$

$$V_1 = -E_1 + I_1 z_1$$



$$V_2 = E_2 - I_2 R_2 - I_2 j X_2$$

$$= E_2 - I_2 [R_2 + j X_2]$$

$$V_2 = E_2 - I_2 Z_2$$

Q. The primary of a 1000/250V transformer has a resistance of  $0.15 \Omega$  and leakage reactance of  $0.8 \Omega$ . Find the primary induced EMF when primary current is 60 A and 0.8 power factor lag.

A. Given:

$$V_1 = 1000 \angle 0^\circ \quad V_2 = 250 \angle 0^\circ$$

$$R = 0.15 \Omega$$

$$\cos \phi = 0.8$$

$$\phi = 36.87^\circ$$

$$\therefore I_1 = 60 \angle -36.9^\circ$$

(- sign since current  $\phi$  is lagging)

$$R + jX = Z$$

$$Z = 0.15 + 0.8j$$

$$Z = 0.814 \angle 79.38^\circ \Omega$$

$$V_1 = -E_1 + I_1 Z_1$$

$$-E_1 = V_1 - I_1 Z_1$$

$$= 1000 \angle 0^\circ - [(60 \angle -36.87^\circ) \times (0.814 \angle 79.38^\circ)]$$

$$-E_1 = 963.99 + j - 33.002$$

$$-E_1 = 964.561 \angle -1.96^\circ$$



$$E_1 = 964.561 \angle 178.04$$

5/10/17  
Q.

The voltage on 2<sup>o</sup> of a single phase transformer is 200V when supplying a load of 8 kW at a power factor of 0.8 lagging. The 2<sup>o</sup> resistance is 0.04  $\Omega$  and 2<sup>o</sup> leakage reactance is 0.8  $\Omega$ .

Calculate induced emf in the 2<sup>o</sup>,  
if kVA we use  $V I$ .

A)

$$V_2 = 200 \angle 0^\circ$$

$$P = 8 \text{ kW}$$

$$\cos \phi = 0.8$$

$$= V I \cos \phi$$

$$\phi = \underline{36.87^\circ}$$

$$R_2 = 0.04$$

$$X_2 = 0.8 \Omega$$

$$P_2 = V_2 I_2 \cos \phi$$

$$\cancel{8000} \quad I_2 = \frac{P_2}{V_2 \cos \phi} = \frac{8000}{200 \times 0.8} = \underline{50 \text{ A}}$$

$$I_2 = 50 \angle -36.8^\circ$$

$$R + jX = Z$$

$$Z = 0.04 + j0.8$$

$$Z = 0.8 \angle 87.18^\circ \Omega$$

$$V_2 = E_2 - I_2 Z_2$$

$$E_2 = V_2 + I_2 Z_2$$

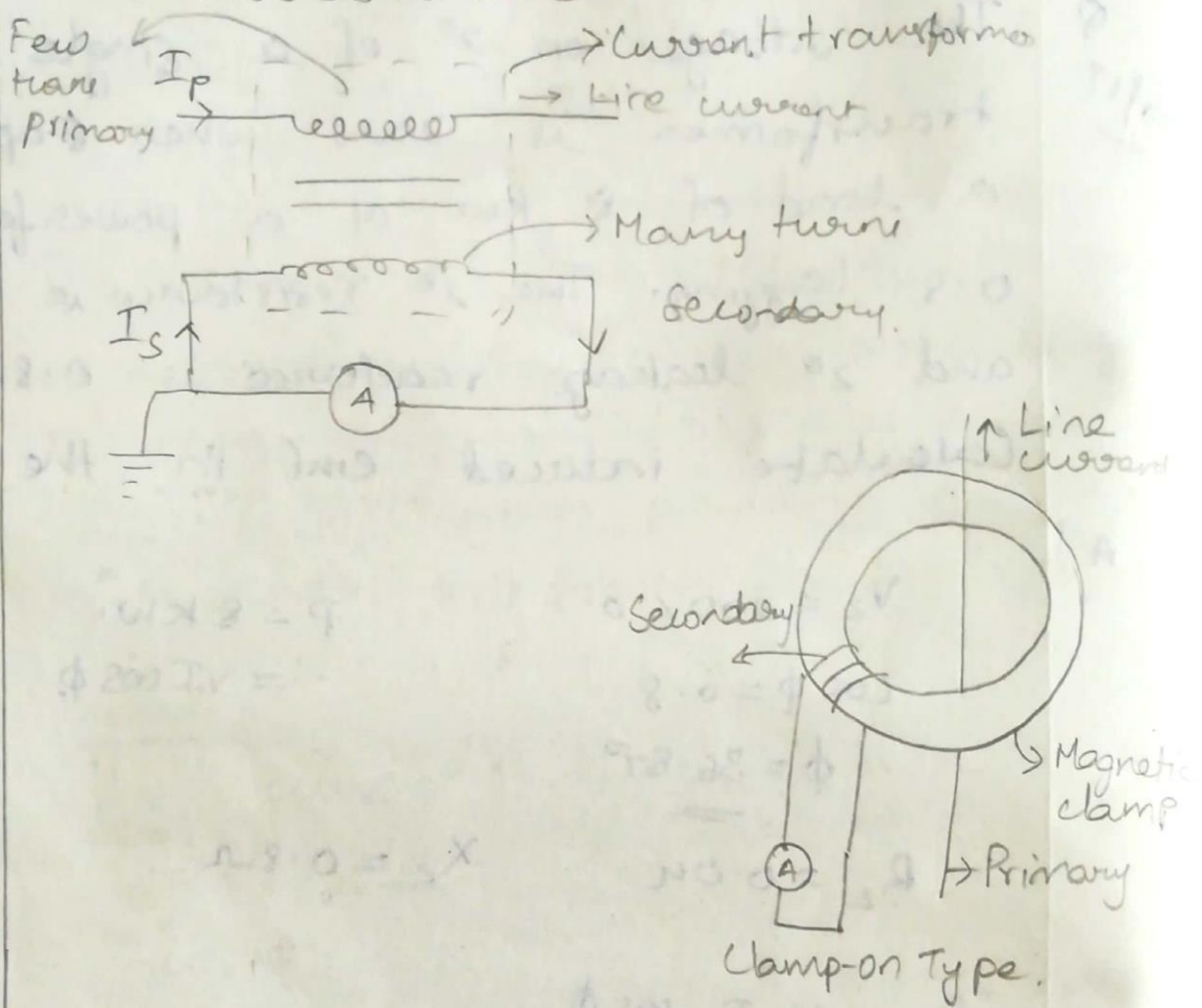
$$= 200 \angle 0^\circ + (50 \angle -36.8^\circ \times 0.8 \angle 87.18^\circ)$$

$$= 227.65 \angle 7.78^\circ \text{ V}$$



# Instrument transformer

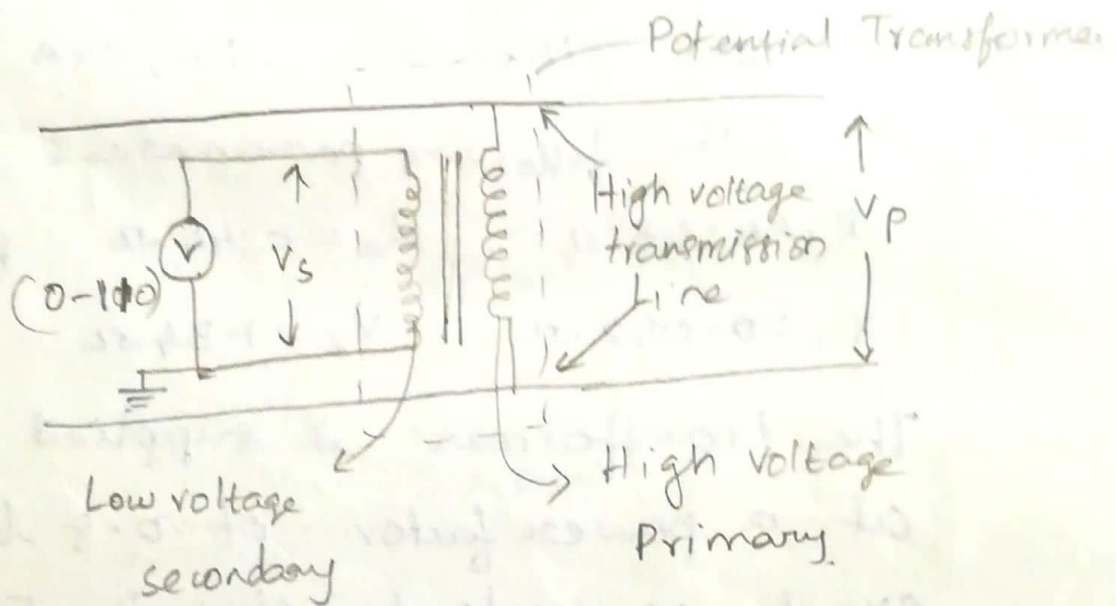
## ① Current transformer (CT)



The current transformer is used to measure high alternative current in a power system. The primary of transformer has few turns of thick wire whereas secondary has many turns of fine wire. The primary of C.T is connected in series with the line whose current is to be measured. The secondary of the transformer is connected across a low range (0-5A) AC ammeter. The line current  $I_p$  and AC ammeter current  $I_s$  are related as  $N_p I_p = N_s I_s$  i.e.  $I_p / I_s = N_s / N_p$  where  $I_p / I_s$  is called

C-T ratio. or current transformation ratio  
 $\therefore$  to find  $I_p = I_c \times \text{C-T ratio}$ .  
 Thus if the reading of AC ammeter is  
 one ampere and current transformation  
 ratio is 100:1, then line current is  
 given by  $1 \times 100 = 100\text{A}$ .

## ② Potential transformer (P.T)



The potential transformer is used to measure high alternating potential difference (voltage) in a power system. The primary of this transformer has many turns, while secondary has few turns. The primary of potential transformer is connected across high voltage line whose voltage is to be measured. A low range (0-110V) AC voltmeter is connected across the secondary. The <sup>line</sup> voltage  $V_p$  and AC voltmeter reading  $V_s$  are related as  

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} \text{ where } \frac{N_p}{N_s} \text{ is}$$



called P.T ratio or potential transformer

$$V_p = \text{P.T ratio} \times V_s$$

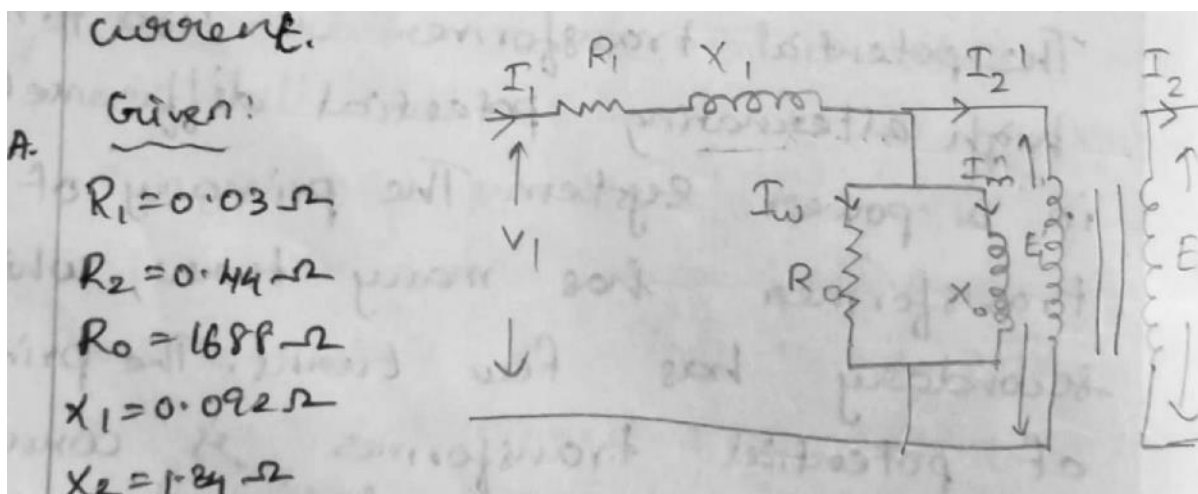
It is a transformer with a ratio of 1000:1

A 1500 V, 16000 V, 1500 kVA, 50 Hz transformer has the following parameters

ratio is 100

$$R_1 = 0.03 \Omega \quad X_2 = 1.34 \Omega \quad X_0 = 256 \Omega$$

The transformer is supplied full load at a power factor of 0.8 lagging using



$$V_2 = 16000$$

0.8 lagging

$$I_2 = 93.75 \angle -36.86^\circ$$

$$\text{Since } \cos \phi = 0.8 \\ \phi = 36.87^\circ$$

$$Z_2 = R_2 + jX_2 \\ = 0.44 + j1.34$$

$$Z_2 = 1.41 \angle 71.82^\circ$$

$$E_2 = V_2 + I_2 Z_2 \\ = 16000 \angle 0^\circ + (93.75 \angle -36.8^\circ \times 1.41 \angle 71.8^\circ)$$

$$E_2 = 16108.52 \angle 0.2693^\circ \text{ V}$$

$$E_2 = 16108.3175 - 72j$$

$$k = \frac{V_2}{V_1} = \frac{16000}{4500} = \underline{\underline{3.56}}$$

$$k = \frac{E_2}{E_1}$$

$$E_1 = \frac{E_2}{k} = \frac{16108.3175 - 72j}{3.56}$$

$$E_1 = 4524.81 + 21.26j$$

$$I_2' = k I_2$$

$$= 93.75 \angle -36.87^\circ \times 3.56$$

$$I_2' = 333.75 \angle -36.87^\circ$$



$$I_m = \frac{E_1}{X_0} \quad I_w = \frac{E_1}{R_0}$$

$$I_m = \frac{4530 \cdot 91 + 21.26j}{256}$$

$$I_m = 17.698 + 0.083j$$

$$I_w = \frac{4530 \cdot 91 + 21.26j}{1688}$$

$$I_w = 2.684 + 0.0125j$$

$$I_1 = I_2' + I_0$$

$$I_0 = I_m + I_w = (17.698 + 0.083j) + (2.684 + 0.0125j) = 20.374 + 0.0956j$$

$$I_1 = I_2' + I_0$$

$$= (333.75 \angle -36.87^\circ) + (20.374 + 0.0956j) = (266.996 - 200.25j) + (20.374 + 0.0956j) = 350.2 \angle -34.85^\circ = 287.39 + -200.11j$$

$$I_w = 2.684 + j0.013A$$

$$I_m = 0.083 - j17.7A$$

$$I_0 = 2.77 - j17.69A$$

$$I_1 = 346.4 \angle -38.94^\circ A$$

## MODULE 4

*Three phase induction motors- slip ring and squirrel cage types- principles of operation – rotating magnetic field- torque slip characteristics- no load and blocked rotor tests. Circle diagrams- methods of starting – direct online – auto transformer starting*

### Three Phase Induction Motors

The three-phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control.

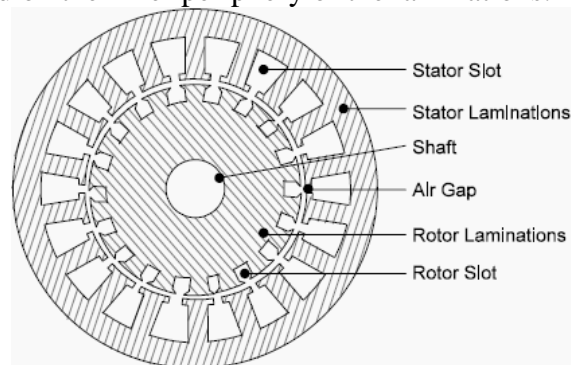
Like any electric motor, a 3-phase induction motor has a stator and a rotor. The stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding). Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name. The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a “transformertype” a.c. machine in which electrical energy is converted into mechanical energy.

#### Construction

A 3-phase induction motor has two main parts (i) stator and (ii) rotor. The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor.

#### Stator

It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations.



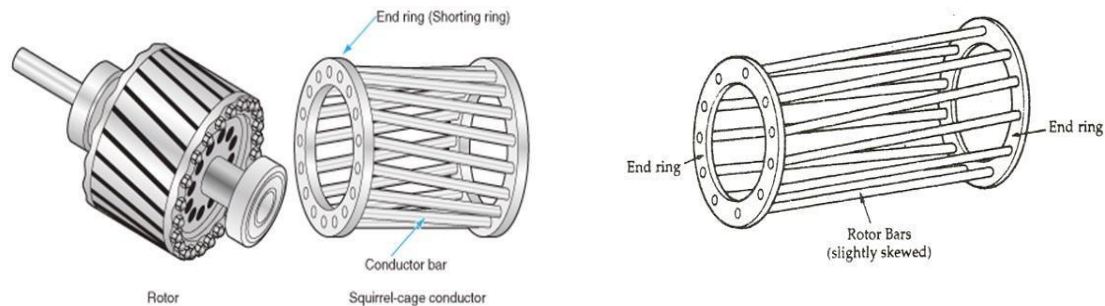
The insulated windings connected to form a balanced 3-phase star or delta connected circuit. The 3-phase stator winding is wound for a definite number of poles as per requirement of speed. Greater the number of poles, lesser is the speed of the motor and vice-versa. When 3-phase supply is given to the stator winding, a rotating magnetic field of constant magnitude is produced. This rotating field induces currents in the rotor by electromagnetic induction.

#### Rotor

The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types:

- (i) Squirrel cage type
- (ii) Wound type

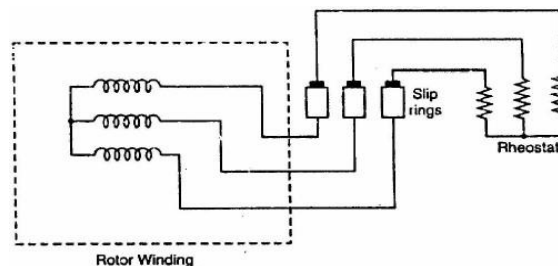
(i) **Squirrel cage rotor.** It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminium bar is placed in each slot. All these bars are joined at each end by metal rings called end rings.



This forms a permanently short-circuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator.

Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of 3-phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque.

(ii) **Wound rotor.** It consists of a laminated cylindrical core and carries a 3-phase winding, similar to the one on the stator. The rotor winding is uniformly distributed in the slots and is usually star-connected. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The three brushes are connected to a 3-phase star-connected rheostat as shown in Fig.

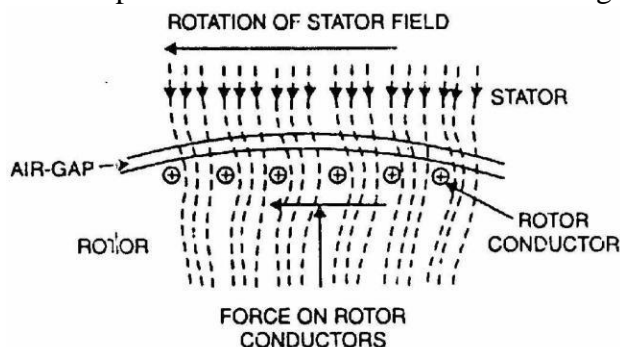


At starting, the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed. The external resistances are used during starting period only. When the motor attains normal speed, the three brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.



### Principle of Operation of 3 phase IM

Consider a portion of 3-phase induction motor as shown in Fig.



- (i) When 3-phase stator winding is energized from a 3-phase supply, a rotating magnetic field is set up which rotates round the stator at synchronous speed  $N_s (= 120 f/P)$ .
- (ii) The rotating field passes through the air gap and cuts the rotor conductors, which as yet, are stationary. Due to the relative speed between the rotating flux and the stationary rotor, e.m.f.s are induced in the rotor conductors. Since the rotor circuit is short-circuited, currents start flowing in the rotor conductors.
- (iii) The current-carrying rotor conductors are placed in the magnetic field produced by the stator. Consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which tends to move the rotor in the same direction as the rotating field.
- (iv) The fact that rotor is urged to follow the stator field (i.e., rotor moves in the direction of stator field) can be explained by Lenz's law. According to this law, the direction of rotor currents will be such that they tend to oppose the cause producing them. Now, the cause producing the rotor currents is the relative speed between the rotating field and the stationary rotor conductors. Hence to reduce this relative speed, the rotor starts running in the same direction as that of stator field and tries to catch it.

### Slip

In Induction motor the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed ( $N$ ) is always less than the stator field speed ( $N_s$ ). This difference in speed depends upon load on the motor.

The difference between the synchronous speed  $N_s$  of the rotating stator field and the actual rotor speed  $N$  is called slip. It is usually expressed as a percentage of synchronous speed i.e.,

$$\% \text{ age slip, } s = \frac{N_s - N}{N_s} \times 100$$

- (i) The quantity  $N_s - N$  is sometimes called slip speed.
- (ii) When the rotor is stationary (i.e.,  $N = 0$ ), slip,  $s = 1$  or 100 %.
- (iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

### Rotor Torque

The torque  $T$  developed by the rotor is directly proportional to:

- (i) rotor current
- (ii) rotor e.m.f.
- (iii) power factor of the rotor circuit

$$\begin{aligned} \therefore T &\propto E_2 I_2 \cos \phi_2 \\ \text{or } T &= K E_2 I_2 \cos \phi_2 \\ &\text{where } I_2 = \text{rotor current at standstill} \\ &\quad E_2 = \text{rotor e.m.f. at standstill} \\ &\quad \cos \phi_2 = \text{rotor p.f. at standstill} \end{aligned}$$

**Note.** The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions.

### Starting Torque (Ts)

Let  $E_2$  = rotor e.m.f. per phase at standstill

$X_2$  = rotor reactance per phase at standstill

$R_2$  = rotor resistance per phase

$$\text{Rotor impedance/phase, } Z_2 = \sqrt{R_2^2 + X_2^2} \quad \dots \text{at standstill}$$

$$\text{Rotor current/phase, } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \quad \dots \text{at standstill}$$

$$\text{Rotor p.f., } \cos \phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \quad \dots \text{at standstill}$$

$$\begin{aligned} \therefore \text{ Starting torque, } T_s &= K E_2 I_2 \cos \phi_2 \\ &= K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \\ &= \frac{K E_2^2 R_2}{R_2^2 + X_2^2} \end{aligned}$$

Generally, the stator supply voltage  $V$  is constant so that flux per pole  $\phi$  set up by the stator is also fixed. This in turn means that e.m.f.  $E_2$  induced in the rotor will be constant.

$$\therefore T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} = \frac{K_1 R_2}{Z_2^2}$$

where  $K_1$  is another constant.

It is clear that the magnitude of starting torque would depend upon the relative values of  $R_2$  and  $X_2$  i.e., rotor resistance/phase and standstill rotor reactance/phase.

It can be shown that  $K = 3/2 \pi N_s$ .

$$\therefore T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Note that here  $N_s$  is in r.p.s.

### Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

$$\text{Now } T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} \quad (i)$$

Differentiating eq. (i) w.r.t.  $R_2$  and equating the result to zero, we get,

$$\frac{dT_s}{dR_2} = K_1 \left[ \frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

$$\text{or } R_2^2 + X_2^2 = 2R_2^2$$

$$\text{or } R_2 = X_2$$

Hence starting torque will be maximum when:

Rotor resistance/phase = Standstill rotor reactance/phase

Under the condition of maximum starting torque,  $\phi_2 = 45^\circ$  and rotor power factor is 0.707 lagging

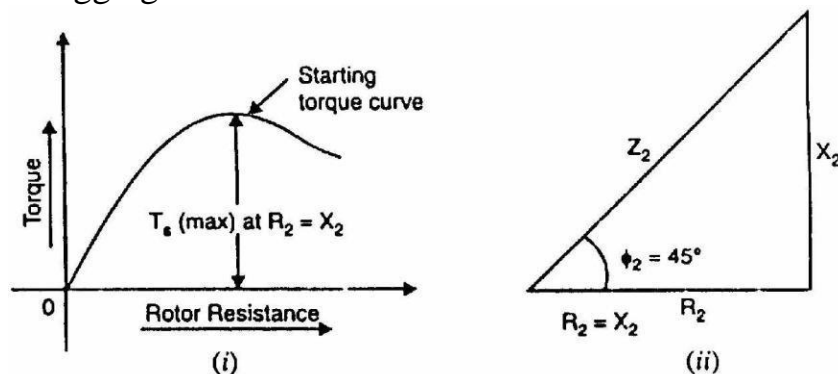


Fig. shows the variation of starting torque with rotor resistance. As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when  $R_2 = X_2$ . If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.

### Torque Under Running Conditions

Let the rotor at standstill have per phase induced e.m.f.  $E_2$ , reactance  $X_2$  and resistance  $R_2$ . Then under running conditions at slip  $s$ ,



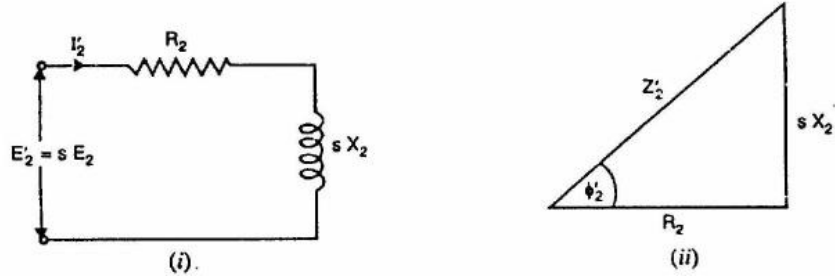
Rotor e.m.f./phase,  $E'_2 = sE_2$

Rotor reactance/phase,  $X'_2 = sX_2$

Rotor impedance/phase,  $Z'_2 = \sqrt{R_2^2 + (sX_2)^2}$

Rotor current/phase,  $I'_2 = \frac{E'_2}{Z'_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$

Rotor p.f.,  $\cos \phi'_m = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$



Running Torque,  $T_r \propto E'_2 I'_2 \cos \phi'_2$

$$\propto \phi I'_2 \cos \phi'_2$$

$$(\because E'_2 \propto \phi)$$

$$\propto \phi \times \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (s X_2)^2}}$$

$$\propto \frac{\phi s E_2 R_2}{R_2^2 + (s X_2)^2}$$

$$= \frac{K \phi s E_2 R_2}{R_2^2 + (s X_2)^2}$$

$$= \frac{K_1 s E_2^2 R_2}{R_2^2 + (s X_2)^2} \quad (\because E_2 \propto \phi)$$

If the stator supply voltage  $V$  is constant, then stator flux and hence  $E_2$  will be constant.

$$\therefore T_r = \frac{K_2 s R_2}{R_2^2 + (s X_2)^2}$$

where  $K_2$  is another constant.

It may be seen that running torque is:

- (i) directly proportional to slip i.e., if slip increases (i.e., motor speed decreases), the torque will increase and vice-versa.
- (ii) directly proportional to square of supply voltage ( $\because E_2 \propto V$ ).

It can be shown that value of  $K_1 = 3/2 \pi N_s$  where  $N_s$  is in r.p.s.

$$\therefore T_r = \frac{3}{2\pi N_s} \cdot \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} = \frac{3}{2\pi N_s} \cdot \frac{s E_2^2 R_2}{(Z'_2)^2}$$

At starting,  $s = 1$  so that starting torque is

$$T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

### Maximum Torque under Running Conditions

$$T_r = \frac{K_2 s R_2}{R_2^2 + s^2 X_2^2} \quad (i)$$

In order to find the value of rotor resistance that gives maximum torque under running conditions, differentiate exp. (i) w.r.t.  $s$  and equate the result to zero i.e.,

$$\frac{dT_r}{ds} = \frac{K_2 [R_2 (R_2^2 + s^2 X_2^2) - 2s X_2^2 (s R_2)]}{(R_2^2 + s^2 X_2^2)^2} = 0$$

or  $(R_2^2 + s^2 X_2^2) - 2s X_2^2 = 0$

or  $R_2^2 = s^2 X_2^2$

or  $R_2 = s X_2$

Thus for maximum torque ( $T_m$ ) under running conditions :

Rotor resistance/phase = Fractional slip  $\times$  Standstill rotor reactance/phase

Now  $T_r \propto \frac{s R_2}{R_2^2 + s^2 X_2^2}$  ... from exp. (i) above

For maximum torque,  $R_2 = s X_2$ . Putting  $R_2 = s X_2$  in the above expression, the maximum torque  $T_m$  is given by;

$$T_m \propto \frac{1}{2 X_2}$$

Slip corresponding to maximum torque,  $s = R_2/X_2$ .

It can be shown that:

$$T_m = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2 X_2} \text{ N - m}$$

It is evident from the above equations that:

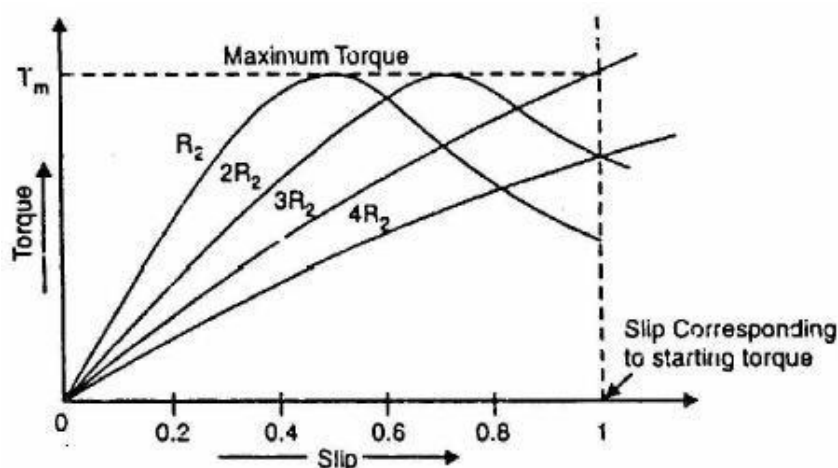
- (i) The value of rotor resistance does not alter the value of the maximum torque but only the value of the slip at which it occurs.
- (ii) The maximum torque varies inversely as the standstill reactance. Therefore, it should be kept as small as possible.
- (iii) The maximum torque varies directly with the square of the applied voltage.
- (iv) To obtain maximum torque at starting ( $s = 1$ ), the rotor resistance must be made equal to rotor reactance at standstill.

### Torque-Slip Characteristics

The motor torque under running conditions is given by;

$$T = \frac{K_2 s R_2}{R_2^2 + s^2 X_2^2}$$

If a curve is drawn between the torque and slip for a particular value of rotor resistance  $R_2$ , the graph thus obtained is called torque-slip characteristic. Fig. shows a family of torque-slip characteristics for a slip-range from  $s = 0$  to  $s = 1$  for various values of rotor resistance.



The following points may be noted carefully:

- (i) At  $s = 0$ ,  $T = 0$  so that torque-slip curve starts from the origin.
- (ii) At normal speed, slip is small so that  $s X_2$  is negligible as compared to  $R_2$ .

$$\therefore T \propto s / R_2$$

$$\propto s \dots \text{as } R_2 \text{ is constant}$$

Hence torque slip curve is a straight line from zero slip to a slip that corresponds to full-load.

- (iii) As slip increases beyond full-load slip, the torque increases and becomes maximum at  $s = R_2/X_2$ . This maximum torque in an induction motor is called pull-out torque or break-down torque. Its value is at least twice the full-load value when the motor is operated at rated voltage and frequency.



(iv) When slip increases beyond that corresponding maximum torque, the term  $s^2 X_2^2$  increases very rapidly so that  $R_2^2$  may be neglected as compared to  $s^2 X_2^2$ .

$$\begin{aligned} \therefore T &\propto s / s^2 X_2^2 \\ &\propto 1/s \dots \quad \text{as } X_2^2 \text{ is constant} \end{aligned}$$

Thus the torque is now inversely proportional to slip. Hence torque-slip curve is a rectangular hyperbola.

(v) The maximum torque remains the same and is independent of the value of rotor resistance. Therefore, the addition of resistance to the rotor circuit does not change the value of maximum torque but it only changes the value of slip at which maximum torque occurs.

### Methods of Starting 3-Phase Induction Motors

The method to be employed in starting a given induction motor depends upon the size of the motor and the type of the motor. The common methods used to start induction motors are:

- (i) Direct-on-line starting
- (ii) Stator resistance starting
- (iii) Autotransformer starting
- (iv) Star-delta starting
- (v) Rotor resistance starting

Methods (i) to (iv) are applicable to both squirrel-cage and slip ring motors. However, method (v) is applicable only to slip ring motors. In practice, any one of the first four methods is used for starting squirrel cage motors, depending upon the size of the motor. But slip ring motors are invariably started by rotor resistance starting.

#### Methods of Starting Squirrel-Cage Motors

Except direct-on-line starting, all other methods of starting squirrel-cage motors employ reduced voltage across motor terminals at starting.

#### (i) Direct-on-line starting

This method of starting is just what the name implies—the motor is started by connecting it directly to 3-phase supply. The impedance of the motor at standstill is relatively low and when it is directly connected to the supply system, the starting current will be high (4 to 10 times the full-load current) and at a low power factor. Consequently, this method of starting is suitable for relatively small (up to 7.5 kW) machines.

#### Relation between starting and F.L. torques.

We know that:

$$\text{Rotor input} = 2\pi N_s T = kT$$

But Rotor Cu loss =  $s \times$  Rotor input

$$3(I_2')^2 R_2 = s kT$$

$$\text{or} \quad T \propto (I_2')^2 / s$$

or  $T \propto I_1^2/s$  ( $\because I_2 \propto I_1$ )

If  $I_{st}$  is the starting current, then starting torque ( $T_{st}$ ) is

$$T \propto I_{st}^2 \quad (\because \text{at starting } s=1)$$

If  $I_f$  is the full-load current and  $s_f$  is the full-load slip, then,

$$T_f \propto I_f^2/s_f$$

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f$$

When the motor is started direct-on-line, the starting current is the short-circuit (blocked-rotor) current  $I_{sc}$ .

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

Let us illustrate the above relation with a numerical example. Suppose  $I_{sc} = 5 I_f$  and full-load slip  $s_f = 0.04$ . Then,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f = \left(\frac{5 I_f}{I_f}\right)^2 \times 0.04 = (5)^2 \times 0.04 = 1$$

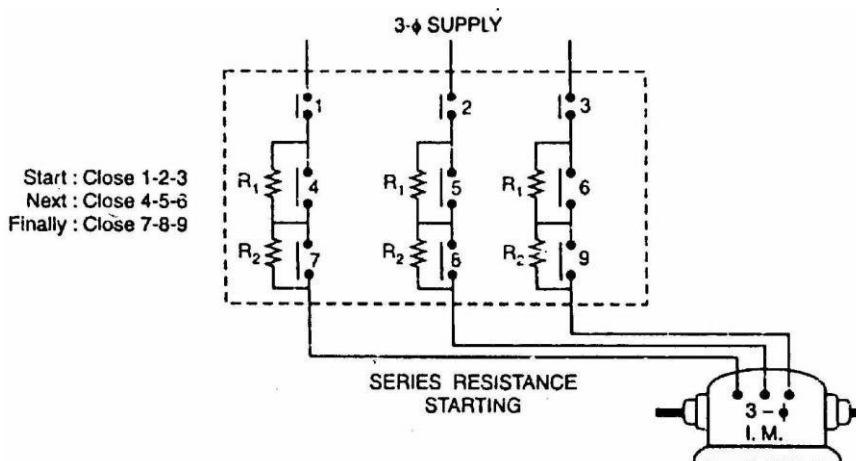
$$\therefore T_{st} = T_f$$

Note that starting current is as large as five times the full-load current but starting torque is just equal to the full-load torque. Therefore, starting current is very high and the starting torque is comparatively low. If this large starting current flows for a long time, it may overheat the motor and damage the insulation.

### (ii) Stator resistance starting

In this method, external resistances are connected in series with each phase of stator winding during starting. This causes voltage drop across the resistances so that voltage available across motor terminals is reduced and hence the starting current. The starting resistances are gradually cut out in steps (two or more steps) from the stator circuit as the motor picks up speed. When the motor attains rated speed, the resistances are completely cut out and full line voltage is applied to the rotor.

This method suffers from two drawbacks. First, the reduced voltage applied to the motor during the starting period lowers the starting torque and hence increases the accelerating time. Secondly, a lot of power is wasted in the starting resistances.



**Relation between starting and F.L. torques.** Let  $V$  be the rated voltage/phase. If the voltage is reduced by a fraction  $x$  by the insertion of resistors in the line, then voltage applied to the motor per phase will be  $xV$ .

$$I_{st} = x I_{sc}$$

Now 
$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f$$

or 
$$\frac{T_{st}}{T_f} = x^2 \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

Thus while the starting current reduces by a fraction „ $x$ ” of the rated-voltage starting current ( $I_{sc}$ ), the starting torque is reduced by a fraction „ $x^2$ ” of that obtained by direct switching. The reduced voltage applied to the motor during the starting period lowers the starting current but at the same time increases the accelerating time because of the reduced value of the starting torque. Therefore, this method is used for starting small motors only.

**(iii) Autotransformer starting**

This method also aims at connecting the induction motor to a reduced supply at starting and then connecting it to the full voltage as the motor picks up sufficient speed.

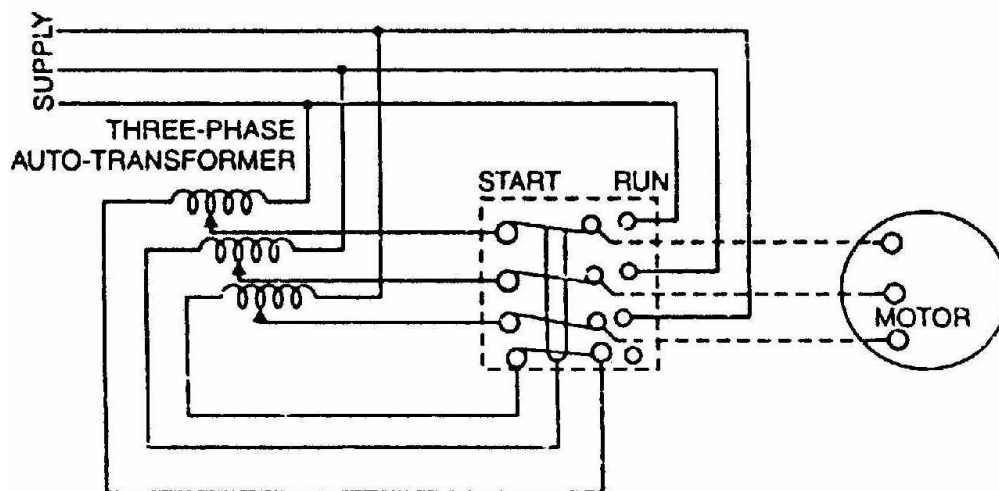




Fig. shows the circuit arrangement for autotransformer starting. The tapping on the autotransformer is so set that when it is in the circuit, 65% to 80% of line voltage is applied to the motor.

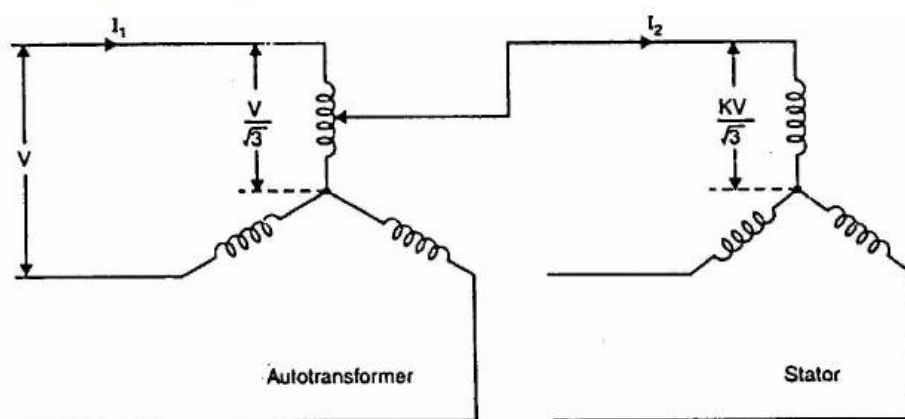
At the instant of starting, the change-over switch is thrown to “start” position. This puts the autotransformer in the circuit and thus reduced voltage is applied to the circuit. Consequently, starting current is limited to safe value.

When the motor attains about 80% of normal speed, the changeover switch is thrown to “run” position. This takes out the autotransformer from the circuit and puts the motor to full line voltage. Autotransformer starting has several advantages viz low power loss, low starting current and less radiated heat. For large machines (over 25 H.P.), this method of starting is often used. This method can be used for both star and delta connected motors.

**Relation between starting And F.L. torques.** Consider a star-connected squirrel-cage induction motor. If  $V$  is the line voltage, then voltage across motor phase on direct switching is  $V/\sqrt{3}$  and starting current is  $I_{st} = I_{sc}$ . In case of autotransformer, if a tapping of transformation ratio  $K$  (a fraction) is used, then phase voltage across motor is  $KV/\sqrt{3}$  and  $I_{st} = K I_{sc}$ ,

$$\text{Now} \quad \frac{T_{st}}{T_f} = \left( \frac{I_{st}}{I_f} \right)^2 \times s_f = \left( \frac{K I_{sc}}{I_f} \right)^2 \times s_f = K^2 \left( \frac{I_{sc}}{I_f} \right)^2 \times s_f$$

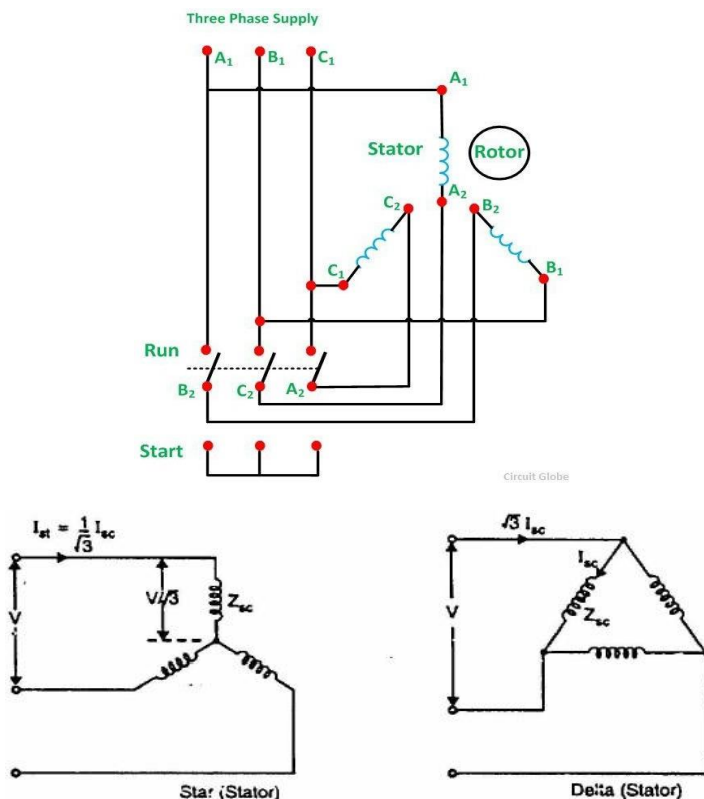
$$\therefore \quad \frac{T_{st}}{T_f} = K^2 \left( \frac{I_{sc}}{I_f} \right)^2 \times s_f$$



The current taken from the supply or by autotransformer is  $I_1 = KI^2 = K^2 I_{sc}$ . Note that motor current is  $K$  times, the supply line current is  $K^2$  times and the starting torque is  $K^2$  times the value it would have been on direct-on-line starting.

#### (iv) Star-delta starting

The stator winding of the motor is designed for delta operation and is connected in star during the starting period. When the machine is up to speed, the connections are changed to delta. The circuit arrangement for star-delta starting is shown in Fig.



The six leads of the stator windings are connected to the changeover switch as shown. At the instant of starting, the changeover switch is thrown to “Start” position which connects the stator windings in star. Therefore, each stator phase gets  $V/\sqrt{3}$  volts where  $V$  is the line voltage. This reduces the starting current. When the motor picks up speed, the changeover switch is thrown to “Run” position which connects the stator windings in delta. Now each stator phase gets full line voltage  $V$ .

**Relation between starting and F.L. torques.** In direct delta starting,

Starting current/phase,  $I_{sc} = V/Z_{sc}$  where  $V$  = line voltage

Starting line current =  $\sqrt{3} I_{sc}$

In star starting, we have,

Starting current/phase,  $I_{st} = \frac{V/\sqrt{3}}{Z_{sc}} = \frac{1}{\sqrt{3}} I_{sc}$

Now 
$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times S_f = \left(\frac{I_{sc}}{\sqrt{3} \times I_f}\right)^2 \times S_f$$

or 
$$\frac{T_{st}}{T_f} = \frac{1}{3} \left(\frac{I_{sc}}{I_f}\right)^2 \times S_f$$

where  $I_{sc}$  = starting phase current (delta)  
 $I_f$  = F.L. phase current (delta)

Note that in star-delta starting, the starting line current is reduced to one-third as compared to starting with the winding delta connected. Further, starting torque is reduced to one-third of that obtainable by direct delta starting. This method is cheap but limited to applications where high starting torque is not necessary e.g., machine tools, pumps etc.

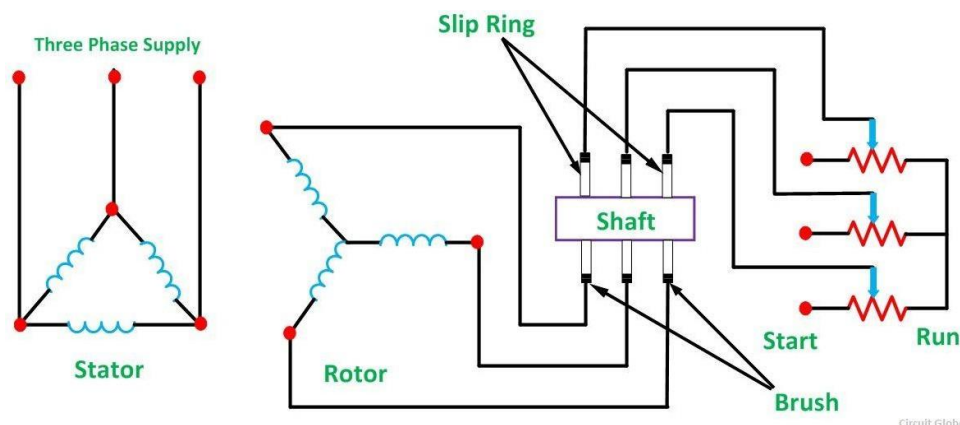
The disadvantages of this method are:

- (a) With star-connection during starting, stator phase voltage is  $1/\sqrt{3}$  times the line voltage. Consequently, starting torque is  $1/3$  or  $1/3$  times the value it would have with  $\Delta$ -connection. This is rather a large reduction in starting torque.
- (b) The reduction in voltage is fixed.

This method of starting is used for medium-size machines (upto about 25 H.P.).

### Starting of Slip-Ring Motors

Slip-ring motors are invariably started by rotor resistance starting. In this method, a variable star-connected rheostat is connected in the rotor circuit through slip rings and full voltage is applied to the stator winding as shown in fig



At starting full starting resistance is connected and thus the supply current to the stator is reduced. The rotor begins to rotate, and the rotor resistances are gradually cut out as the speed of the motor increases. When the motor is running at its rated full load speed, the starting resistances are cut out completely, and the slip rings are short-circuited.

### Slip-Ring Motors Versus Squirrel Cage Motors

The slip-ring induction motors have the following advantages over the squirrel cage motors:

- (i) High starting torque with low starting current.
- (ii) Smooth acceleration under heavy loads.
- (iii) No abnormal heating during starting.
- (iv) Good running characteristics after external rotor resistances are cut out.
- (v) Adjustable speed.



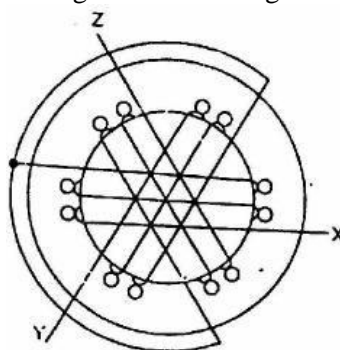
The disadvantages of slip-ring motors are:

- (i) The initial and maintenance costs are greater than those of squirrel cage motors.
- (ii) The speed regulation is poor when run with resistance in the rotor circuit

**Rotating Magnetic Field Due to 3-Phase Currents**

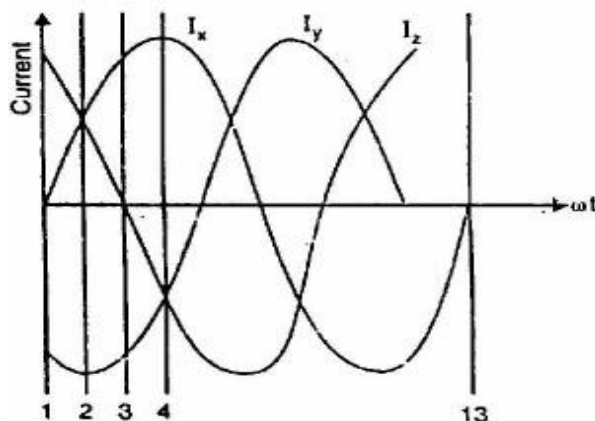
When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do not remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating field. It can be shown that magnitude of this rotating field is constant and is equal to  $1.5 \phi_m$  where  $\phi_m$  is the maximum flux due to any phase.

Consider a 2-pole, 3-phase winding as shown in Fig.



(i)

The three phases X, Y and Z are energized from a 3-phase source and currents in these phases are indicated as  $I_x$ ,  $I_y$  and  $I_z$



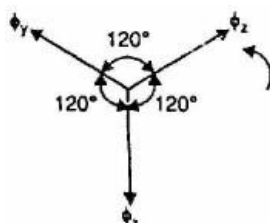
(ii)

The fluxes produced by these currents are given by:

$$\phi_x = \phi_m \sin \omega t$$

$$\phi_y = \phi_m \sin (\omega t - 120^\circ)$$

$$\phi_z = \phi_m \sin (\omega t - 240^\circ)$$

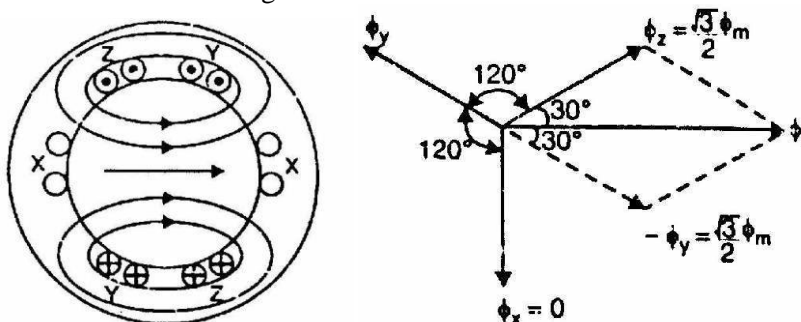


Here  $\phi_m$  is the maximum flux due to any phase. Above Fig. shows the phasor diagram of the three fluxes.

**Proof of this 3-phase supply produces a rotating field of constant magnitude equal to  $1.5 \phi_m$ .**

1. At instant 1 [See Fig. (ii) and below Fig.]

The current in phase X is zero and currents in phases Y and Z are equal and opposite. The currents are flowing outward in the top conductors and inward in the bottom conductors. This establishes a resultant flux towards right.



The magnitude of the resultant flux is constant and is equal to  $1.5 \phi_m$

At instant 1,  $\omega t = 0^\circ$ . Therefore, the three fluxes are given by;

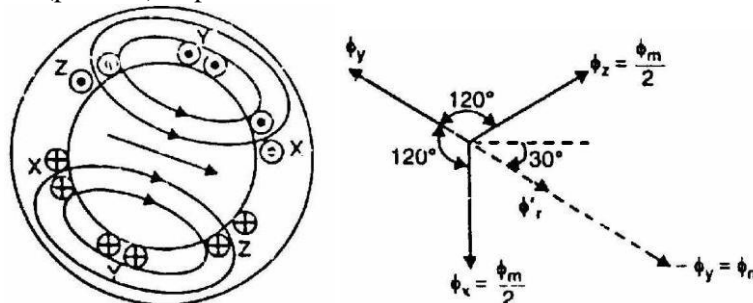
$$\phi_x = 0; \quad \phi_y = \phi_m \sin(-120^\circ) = -\frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_z = \phi_m \sin(-240^\circ) = \frac{\sqrt{3}}{2}\phi_m$$

The phasor sum of  $-\phi_y$  and  $\phi_z$  is the resultant flux  $\phi_r$  [See Fig. (8.7)]. It is clear that:

$$\text{Resultant flux, } \phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos \frac{60^\circ}{2} = 2 \times \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2} = 1.5 \phi_m$$

2. At instant 2, [See Fig. (ii) and below Fig.] the current is maximum (negative) in  $\phi_y$  phase Y and 0.5 maximum (positive) in phases X and Z.



The magnitude of resultant flux is  $1.5 \phi_m$

At instant 2,  $\omega t = 30^\circ$ . Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 30^\circ = \frac{\phi_m}{2}$$

$$\phi_y = \phi_m \sin(-90^\circ) = -\phi_m$$

$$\phi_z = \phi_m \sin(-210^\circ) = \frac{\phi_m}{2}$$

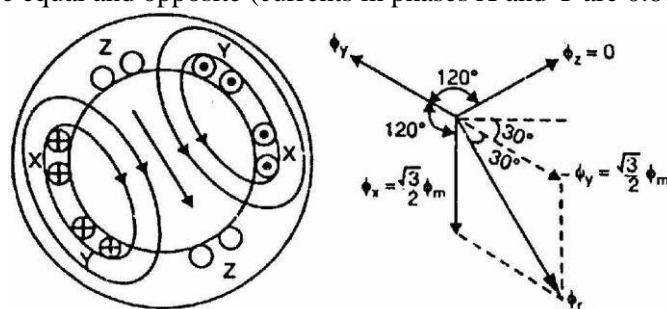
The phasor sum of  $\phi_x$ ,  $-\phi_y$  and  $\phi_z$  is the resultant flux  $\phi_r$

$$\text{Phasor sum of } \phi_x \text{ and } \phi_z, \phi'_r = 2 \times \frac{\phi_m}{2} \cos \frac{120^\circ}{2} = \frac{\phi_m}{2}$$

$$\text{Phasor sum of } \phi'_r \text{ and } -\phi_y, \phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m$$

Note that resultant flux is displaced  $30^\circ$  clockwise from position 1.

3. At instant 3, [See Fig. (ii) and below Fig.] current in phase Z is zero and the currents in phases X and Y are equal and opposite (currents in phases X and Y are  $0.866 \times \text{max. value}$ ).



The magnitude of resultant flux is  $1.5 \phi_m$

At instant 3,  $\omega t = 60^\circ$ . Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 60^\circ = \frac{\sqrt{3}}{2} \phi_m;$$

$$\phi_y = \phi_m \sin(-60^\circ) = -\frac{\sqrt{3}}{2} \phi_m;$$

$$\phi_z = \phi_m \sin(-180^\circ) = 0$$

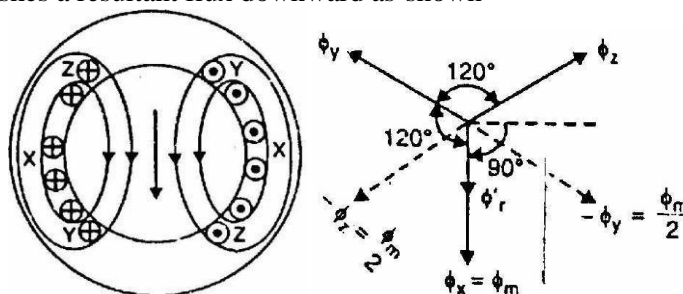
The resultant flux  $\phi_r$  is the phasor sum of  $\phi_x$  and  $-\phi_y$  ( $\because \phi_z = 0$ ).

$$\phi_r = 2 \times \frac{\sqrt{3}}{2} \phi_m \cos \frac{60^\circ}{2} = 1.5 \phi_m$$

Note that resultant flux is displaced  $60^\circ$  clockwise from position 1.

Fig.(8.9)

4. At instant 4, [See Fig. (ii) and below Fig.] the current in phase X is maximum (positive) and the currents in phases Y and Z are equal and negative (currents in phases Y and Z are  $0.5 \times \text{max. value}$ ). This establishes a resultant flux downward as shown



At instant 4,  $\omega t = 90^\circ$ . Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 90^\circ = \phi_m$$

$$\phi_y = \phi_m \sin(-30^\circ) = -\frac{\phi_m}{2}$$

$$\phi_z = \phi_m \sin(-150^\circ) = -\frac{\phi_m}{2}$$

The phasor sum of  $\phi_x$ ,  $-\phi_y$  and  $-\phi_z$  is the resultant flux  $\phi_r$

$$\text{Phasor sum of } -\phi_z \text{ and } -\phi_y, \phi'_r = 2 \times \frac{\phi_m}{2} \cos \frac{120^\circ}{2} = \frac{\phi_m}{2}$$

$$\text{Phasor sum of } \phi'_r \text{ and } \phi_x, \phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m$$

Note that the resultant flux is downward i.e., it is displaced  $90^\circ$  clockwise from position 1.

It follows from the above discussion that a 3-phase supply produces a rotating field of constant value ( $= 1.5 \phi_m$ , where  $\phi_m$  is the maximum flux due to any phase).



### Circle Diagram of Induction Motor

The circle diagram of an induction motor is very useful to study its performance under all operating conditions. The “CIRCLE DIAGRAM” means that it is figure or curve which is drawn has a circular shape. As we know, the diagrammatic representation is easier to understand and remember compared to theoretical and mathematical descriptions.

#### Importance of Circle Diagram

The diagram provides information which is not provided by an ordinary phasor diagram. A phasor diagram gives relation between current and voltage only at a single circuit condition. If the condition changes, we need to draw the phasor diagram again. But a circle diagram may be referred to as a phasor diagram drawn in one plane for more than one circuit conditions. On the context of induction motor, which is our main interest, we can get information about its power output, power factor, torque, slip, speed, copper loss, efficiency etc. in a graphical or in a diagrammatic representation.

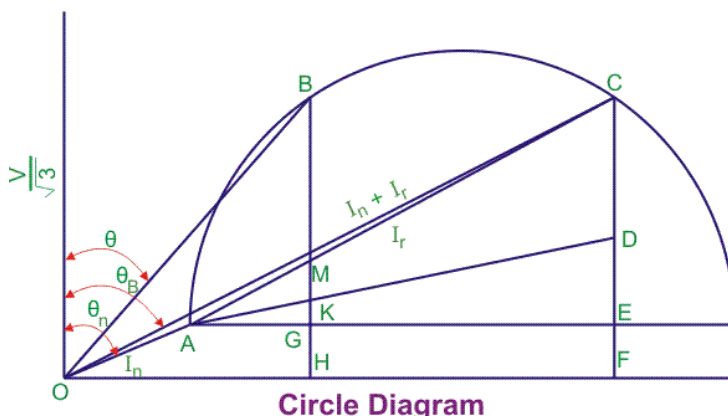
#### Test Performed to Compute Data Required for Drawing Circle Diagram

We have to perform no load and blocked rotor test in an induction motor. In no load test, the induction motor is run at no load and by two watt meter method, its total power consumed is calculated which is composed of no load losses only. Slip is assumed to be zero. From here no load current and the angle between voltage and current required for drawing circle diagram is calculated. The angle will be large as in the no load condition induction motor has high inductive reactance.

#### Procedure to Draw the Circle Diagram

We have to assume a suitable before drawing it. This assumption is done according to our convenience.

1. The no load current and the no load angle calculated from no load test is plotted. This is shown by the line OA, where  $\Theta_0$  is the no load power factor angle.
2. The short circuit current and the angle obtained from block rotor test is plotted. This is shown by the line OC and the angle is shown by  $\Theta_B$ .
3. The right bisector of the line AC is drawn which bisects the line and it is extended to cut in the line AE which gives us the centre.
4. The stator current is calculated from the equivalent circuit of the induction motor which we get from the two tests. That current is plotted in the circle diagram according to the scale with touching origin and a point in the circle diagram which is shown by B.
5. The line AC is called the power line. By using the scale for power conversion that we have taken in the circle diagram, we can get the output power if we move vertically above the line AC to the periphery of the circle. The output power is given by the line MB.
6. The total copper loss is given by the line GM.
7. For drawing the torque line, the total copper loss should be separated to both the rotor copper loss and stator copper loss. The line DE gives the stator copper loss and the line CD gives the rotor copper loss. In this way, the point E is selected.
8. The line AD is known as torque line which gives the torque developed by induction motor.



**Maximum Quantities from Circle Diagram**

**Maximum Output Power**

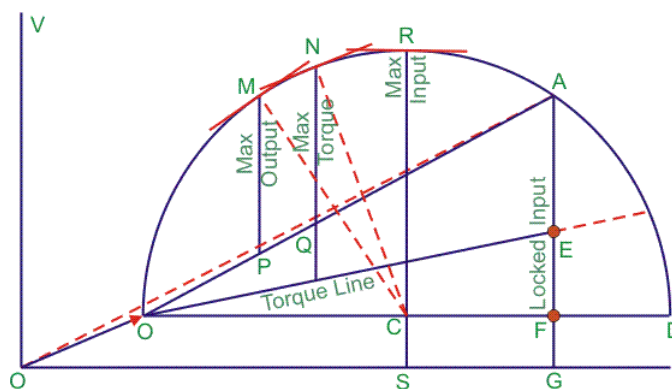
When the tangent to the circle is parallel to the line then output power will be maximum. That point M is obtained by drawing a perpendicular line from the center to the output line and extending it to cut at M.

**Maximum Torque**

When the tangent to the circle is parallel to the torque line, it gives maximum torque. This is obtained by drawing a line from the center in perpendicular to the torque line AD and extending it to cut at the circle. That point is marked as N.

**Maximum Input Power**

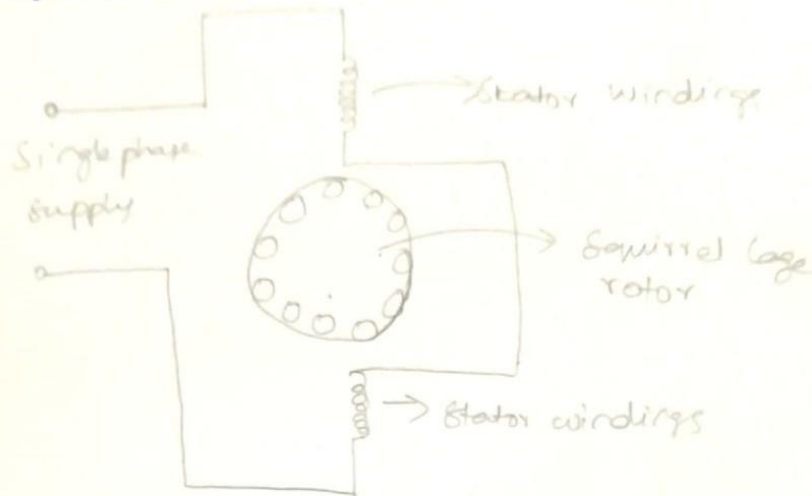
It occurs when tangent to the circle is perpendicular to the horizontal line. The point is the highest point in the circle diagram and drawn to the center and extends up to S. That point is marked as R.



**Conclusion of Circle Diagram**

This method is based on some approximations that we have used in order to draw the circle diagram and also, there is some rounding off of the values as well. So there is some error in this method but it can give good approximate results. Also, this method is very much time consuming so it is drawn at times where the drawing of circle diagram is absolutely necessary. Otherwise, we can go for mathematical formulas or equivalent circuit model in order to find out various parameters.

23/10/17

Single phase motor - Module VSingle phase induction Motor.Principle of Operation of single phase induction motor:

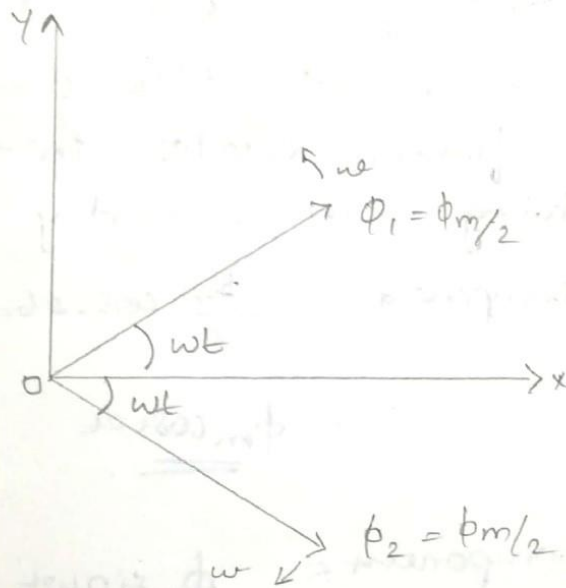
single phase induction motor is not a self starting motor. It require some starting means it has a squirrel cage rotor and a single phase winding on a stator. In single phase stator windings, a magnetic field is produced that pulsate in a sinusoidal manner. The field polarity reverses after each half cycle. But the field does not rotate. In case of a stationary squirrel cage motor, the alternating flux cannot produce rotation and if the rotor of single phase motor is rotated in one direction by some means



it will continue to rotate in the direction of rotation.

### Double field revolving theory:

This theory is proposed to explain no torque at the starting and yet torque once rotated.



This theory is based on the fact that, an alternating sinusoidal flux  $\phi = \phi_m \cos \omega t$  can be represented by two revolving fluxes each equal to half of maximum value of alternating flux i.e.  $\phi_m/2$  and rotating at synchronous speed  $N_s = \frac{120f}{P}$  where  $\omega = 2\pi f$  in opposite directions.

The instantaneous value of flux due to

stator current of single phase induction motor  $\phi = \phi_m \cos \omega t$ .

Consider the rotating magnetic fluxes  $\phi_1$  and  $\phi_2$  each having the magnitude  $\phi_m/2$  rotating in opposite directions with angular velocity  $\omega$ .

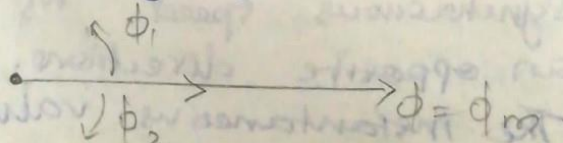
Let the fluxes start rotating from Ox axis at time  $t=0$ . After  $t$  sec the fluxes rotate through an angle  $\omega t$  resolving in x and y directions,

$$\begin{aligned} x \text{ component} &= \frac{\phi_m}{2} \cos \omega t + \frac{\phi_m}{2} \cos \omega t \\ &= \underline{\underline{\phi_m \cos \omega t}} \end{aligned}$$

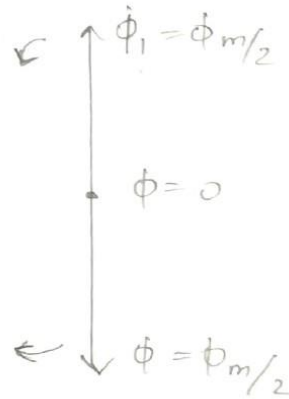
$$\begin{aligned} y \text{ component} &= \phi_1 \sin \omega t - \phi_2 \sin \omega t \\ &= \frac{\phi_m}{2} \sin \omega t - \frac{\phi_m}{2} \sin \omega t \\ &= 0 \end{aligned}$$

∴ The resultant flux  $\phi = \sqrt{(\phi_m \cos \omega t)^2 + 0^2}$   
 $\phi = \phi_m \cos \omega t$  (along x-axis)

When the rotating flux vectors are in phase

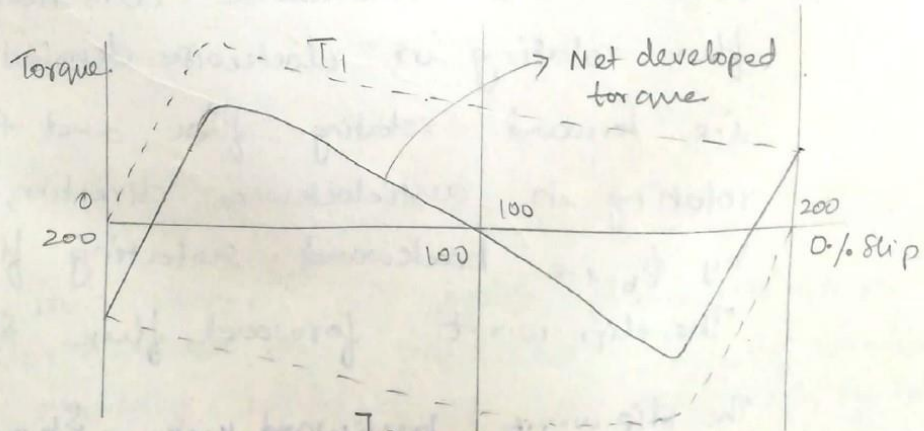
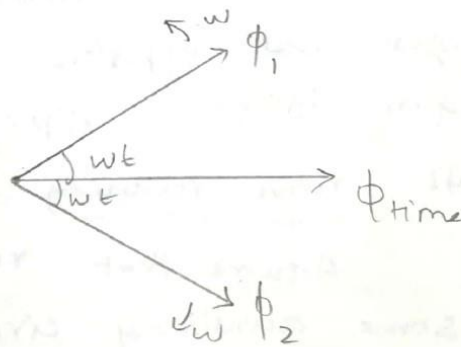


The resultant flux will be  $\phi = \phi_m$  when they are out of phase by an angle  $180^\circ$



Resultant flux  $\phi = 0$ .

Rotor at standstill.





25/10/17

Consider the rotor is stationary and stator winding is connected to a single phase supply. Alternating flux produced by the stator winding can be represented as sum of two rotating fluxes  $\phi_1$  and  $\phi_2$ , each having the value i.e. one half of the maximum values of alternating flux.

rotating with synchronous speed,  $N_s = \frac{120f}{P}$

Let  $\phi_1$  rotate in anticlockwise direction and  $\phi_2$  rotate in clockwise direction,  $\phi_1$

produces torque  $T_1$  and  $\phi_2$  produces

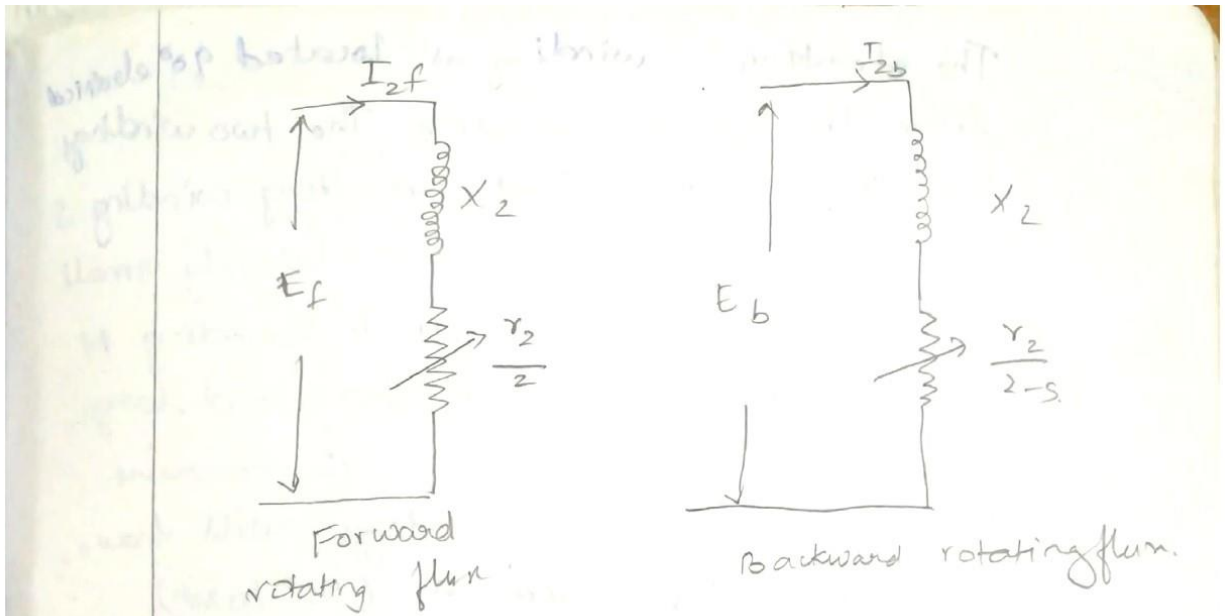
torque  $T_2$ . At standstill condition, these are equal and opposite and the net torque is equal to zero. (slip = 1,  $N = 0$ )

At rotor running

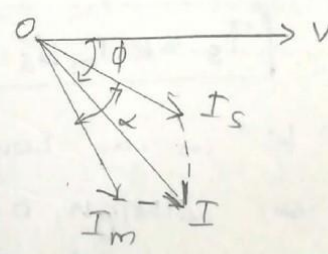
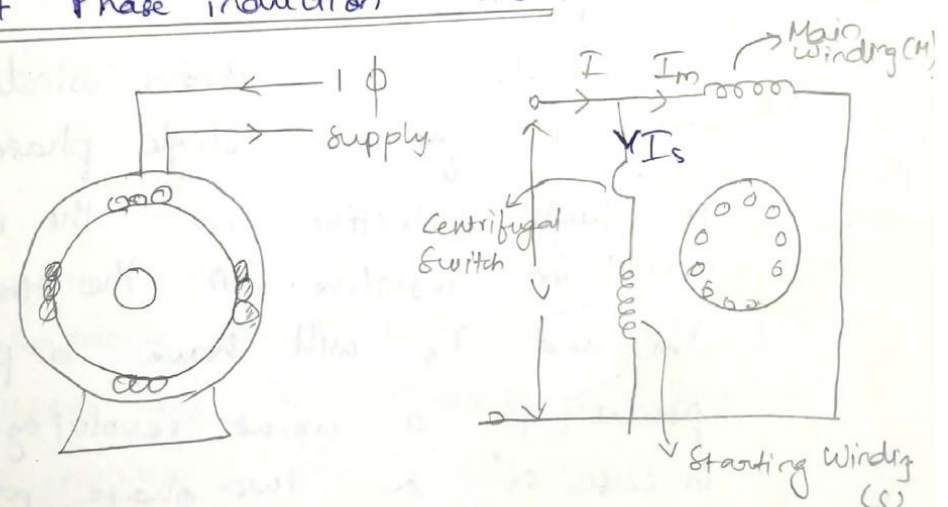
Assume that rotor is started by using some auxiliary circuit. Let the rotating direction be clockwise direction. The flux rotating in clockwise denoted as  $\phi_f$  i.e. forward rotating flux and the flux rotating in anticlockwise direction, represented by  $\phi_b$  i.e. backward rotating flux.

The slip w.r.t forward flux  $S_f = \frac{N_s - N}{N_s} = S$

The slip w.r.t backward flux  $S_b = \frac{N_s - (-N)}{N_s} = \frac{2N_s - N}{N_s} = \underline{\underline{2 - S}}$



Split Phase induction Motor



The stator of split phase induction motor is provided with an auxiliary or starting winding S in addition to the main or running winding M.

The starting winding is located  $90^\circ$  electrical from the main winding. The two windings are designed such that starting winding  $S$  has high resistance and relatively small reactance, while the main winding  $M$  has relatively low resistance and large reactance. As a result the current flowing in the two windings will have a phase difference  $\alpha$  ( $25^\circ$  to  $30^\circ$ )

Working:

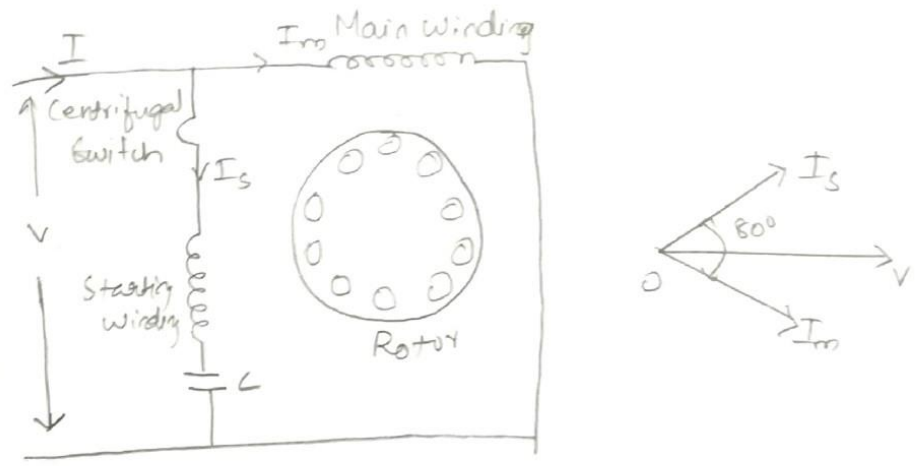
When the two stator windings are energized from a single phase supply due to high inductive in the main winding and high resistive in the starting winding  $I_m$  and  $I_s$  will have a phase angle  $\alpha$  producing a weak revolving field. In case of a two phase machine.

$$T_s = k I_m I_s \sin \alpha \quad \text{— starting torque}$$

where  $k$  is a constant whose magnitude depends on design of the motor. When the motor reaches about 75% of synchronous speed, centrifugal switch opens the circuit of starting winding.



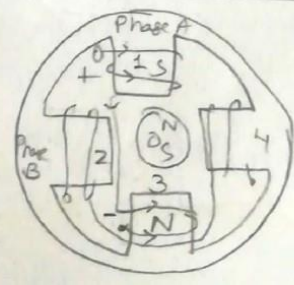
Capacitor Start Motor.



Here a capacitor  $C$  is connected in series with the starting winding. The value of capacitance is chosen such that  $I_s$  leads  $I_m$  about  $80^\circ$  which is greater than the case in split phase motor. Therefore the starting torque  $T_s = k I_m I_s \sin \alpha$  is more. Once the motor reaches 75% of synchronous speed  $N_s$ . The starting winding is open by the centrifugal switch. These motors are used where high starting torque is required.

Stepper Motor

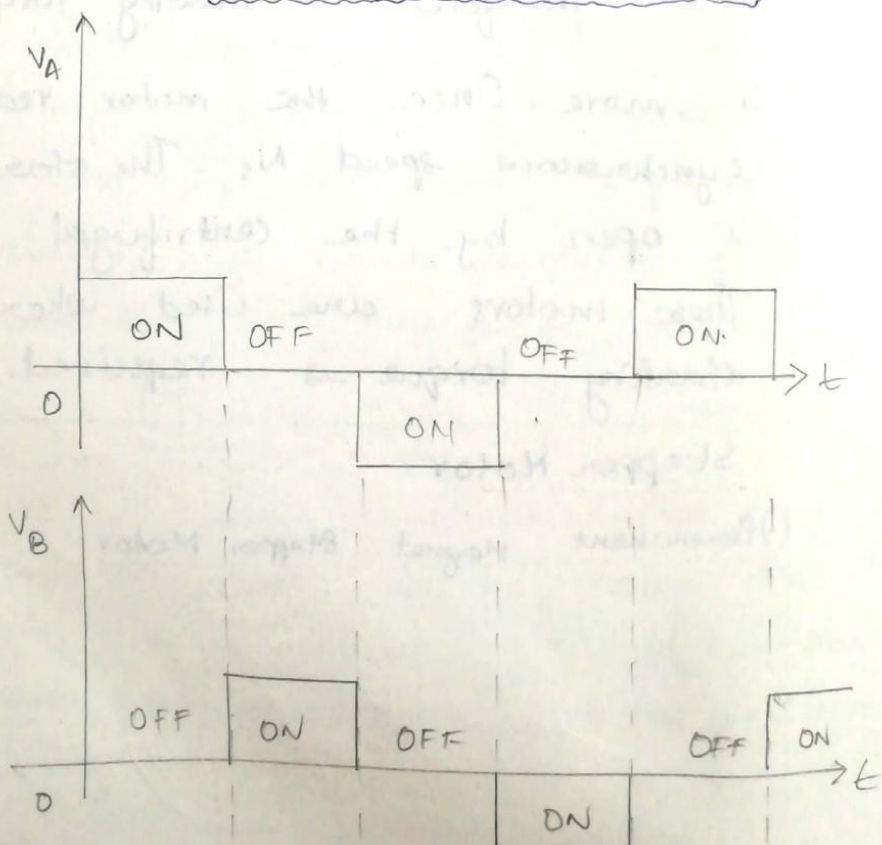
(i) Permanent Magnet Stepper Motor



Truth Table

Cycle	A	B	Position $\delta^\circ$
+	1	0	0
	0	1	90°
-	-1	0	180°
	0	-1	270°
+	1	0	360°

Applied Voltage waveform



Stator windings are energized from a DC source to create two or more stator poles. The rotor of the motor is permanent magnet made of high retentivity of steel alloy. The ~~stator~~ rotor has even no. of poles.

Consider a two phase two pole permanent magnet stepper motor. It has two rotor poles. The phases given are phase A and phase B. For this motor the step angle

$$\alpha = \frac{360^\circ}{m N_r}$$

where  $m$  is no. of phases and

$N_r$  is no. of rotor poles.

∴ The step angle  $\alpha = \frac{360^\circ}{2 \times 2} = 90^\circ$

Case 1:

Only phase A is excited by constant current.

The stator tooth 1 becomes south pole. The north pole of permanent magnet rotor aligned with the south pole. Till the phase A winding remains energized, the rotor will be locked in this position.

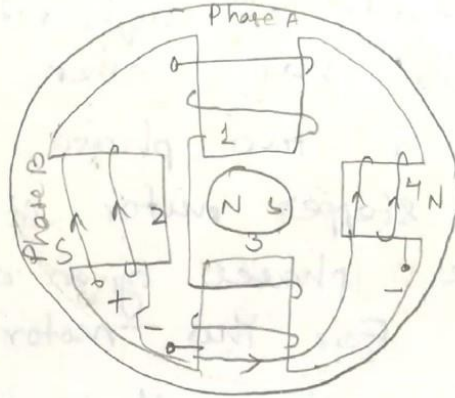
This is represented by first row of truth table.

Now the step angle  $\alpha = 0^\circ$ .



Case 2:

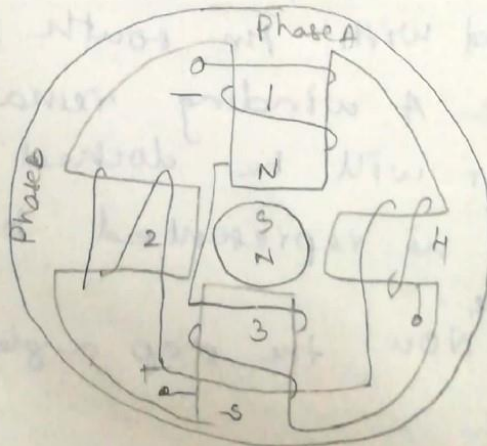
Phase A winding is deenergized and phase B<sup>2</sup> winding is energized.



The stator tooth 2 becomes south pole, the north pole of permanent magnet rotor aligns with the south pole of the stator. Thus the rotor has displaced 90° in anticlockwise direction.

Case 3:

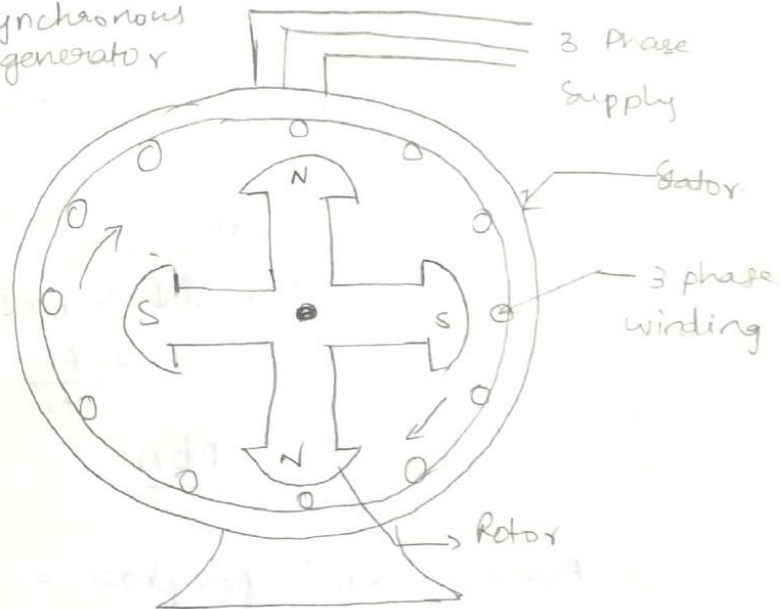
Phase B winding is deenergized and Phase A winding is energized with reverse current.



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The rotor will further rotate  $90^\circ$  in anti-clockwise direction, now the north pole of permanent magnet motor aligns with stator tooth 3.

Alternator / Synchronous generator



Alternator or synchronous generator produces three phase power from the mechanical power. The field poles are placed on the ~~stationary~~ <sup>rotor</sup> part of machine as no commutator is required in an alternator, and the armature winding are on the stator.

Advantages of stationary armature:

- The three phase armature windings placed on the stator have the following advantages:
- (i) Easier to insulate stationary winding for high voltages for which the alternators are usually designed.
  - (ii) Stationary three phase armature can be directly connected to the load.

EMF eqn of Alternator

Let  $Z =$  no. of conductors  
 $\phi =$  flux per pole  
 $P =$  Poles (rotor poles)  $N =$  speed

Induced emf in one stator conductor  
 $= \frac{d\phi}{dt}$

Change in flux  $d\phi = P \times \phi$   
 $dt = \frac{60}{N}$

$\therefore \frac{d\phi}{dt} = \frac{P\phi N}{60}$

Average emf per phase  $= \frac{P\phi N}{60} \times Z \quad \text{--- (1)}$

$N = \frac{120f}{P}$

$\therefore$  (1) becomes  $\frac{P\phi \frac{120f}{P}}{60} \times Z$

Average emf  $= 2f\phi Z$

RMS value of EMF per phase  $= \frac{2f\phi Z \times 1.11}{\sqrt{2}}$

$= \underline{\underline{2.2 f \phi Z}}$  form factor

NOTE:

i) RMS value of EMF per phase = Average value per phase  $\times$  form factor.

Considering pitch factor ( $k_p$ ) and distribution factor of armature winding ( $k_d$ ).



$$\begin{aligned} \text{EMF per phase} &= \frac{2.22}{\sqrt{2}} K_p K_d f \phi z \quad \checkmark \quad z = \frac{Z}{2} \\ &= \frac{4.44}{\sqrt{2}} K_p K_d f \phi T \quad \checkmark \end{aligned}$$

- 1) A 3  $\phi$  50 Hz star connected alternator has 180 conductors per phase and flux per pole is 0.0543 wb. Find EMF generated per phase, emf b/w the terminals, assuming the windings to be full pitched and distribution factor to be 0.96.

Given:  $f = 50 \text{ Hz}$        $Z = 180$        $\phi = 0.0543$

$$K_p = 1$$

$$K_d = 0.96$$

Star Connected

$$\text{Phase } I = \text{Line } I$$

$$\text{Phase } V \neq \text{Line } V$$

$$\text{Line } V = \sqrt{3} \text{ Phase } V$$

Delta Connected

$$\text{Phase } V = \text{Line } V$$

$$\text{Line } I = \sqrt{3} \text{ Phase } I$$

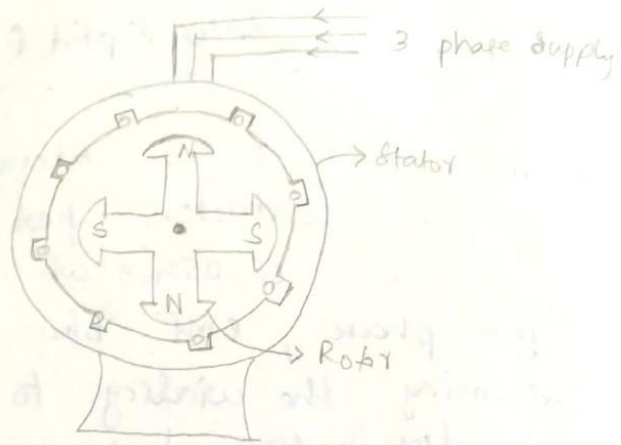
$$\begin{aligned} \text{EMF} &= 2.22 \times 50 \times 180 \times 0.0543 \times 0.96 \\ &= \underline{\underline{1041.52 \text{ V}}} \end{aligned}$$

$$\begin{aligned} \text{EMF b/w terminals} &= \text{Line voltage} \\ &= \sqrt{3} \times \text{EMF} \\ &= \sqrt{3} \times 1041.52 \\ &= \underline{\underline{1803.97 \text{ V}}} \end{aligned}$$

Regulation of alternator  
by emf method.

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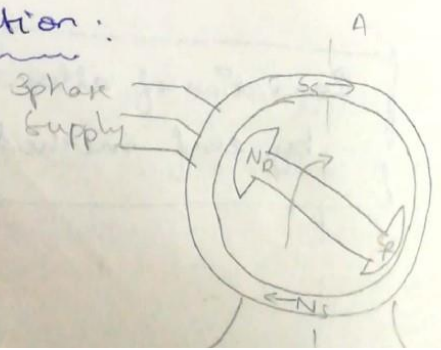
Synchronous Motor

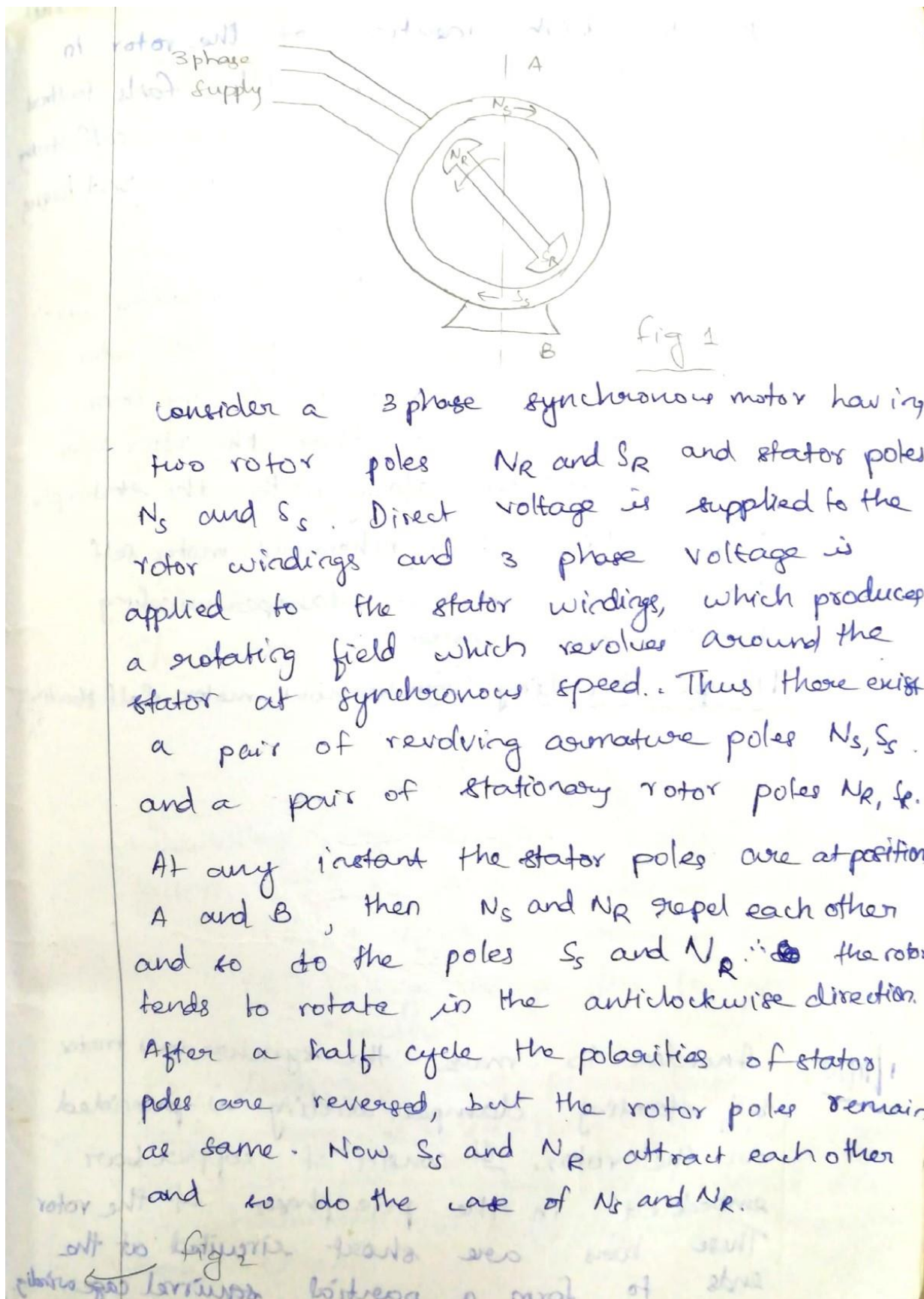


Synchronous motor is a machine that operates at synchronous speed and converts electrical energy to mechanical energy. It has two parts, stator and rotor. Stator has three phase armature windings in the slot of stator core and receives power from a 3-phase supply. The rotor has a set of poles excited by direct current, to form north & south poles. The stator is wound for same no. of poles as the rotor

$$N_s = \frac{120f}{P}$$

Principle of Operation:





Consider a 3 phase synchronous motor having two rotor poles  $N_r$  and  $S_r$  and stator poles  $N_s$  and  $S_s$ . Direct voltage is supplied to the rotor windings and 3 phase voltage is applied to the stator windings, which produces a rotating field which revolves around the stator at synchronous speed. Thus there exist a pair of revolving armature poles  $N_s, S_s$  and a pair of stationary rotor poles  $N_r, S_r$ .

At any instant the stator poles are at position A and B, then  $N_s$  and  $N_r$  repel each other and so do the poles  $S_s$  and  $N_r$ . The rotor tends to rotate in the anticlockwise direction.

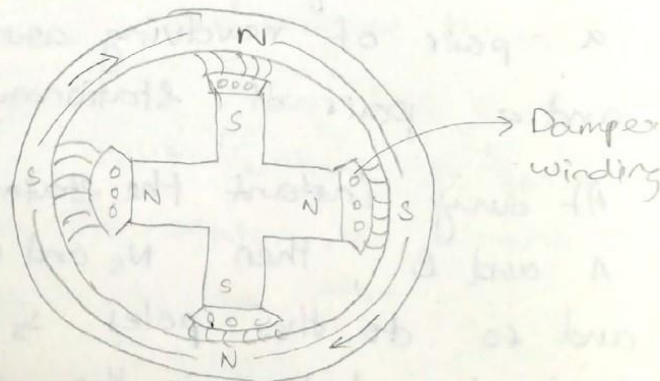
After a half cycle the polarities of stator poles are reversed, but the rotor poles remain the same. Now  $S_s$  and  $N_r$  attract each other and so do the case of  $N_s$  and  $N_r$ .



Due to high inertia of the rotor in both the directions, the motor fails to start. Hence synchronous motor is not a self starting motor. To make continuous unidirectional torque the following can be done.

- (i) Suppose the stator field is rotating in the clockwise direction and the rotor is also rotated in clockwise by some external means such that the rotor poles are interchanged along with the stator poles. For making the synchronous motor self starting, we can use damper winding provided on the rotor.

### Damper Winding (Synchronous motor self starting)



1/11/10

In order to make the synchronous motor self starting damper winding is provided on the rotor. It consists of copper bars embedded in the pole shoes of the rotor. These bars are short circuited at the ends to form a partial squirrel cage winding.

To start with the 3-phase supply is given to the stator winding while the rotor field winding is unenergized. The rotating stator field induces current in the damper winding and the motor starts as an induction motor. When the motor approaches synchronous speed, the rotor is excited with direct current and the resulting poles on the rotor phase pole of opposite polarity on the stator and a strong magnetic attraction is set up b/w them. Thus the rotor revolves at the same speed as the stator field.

Voltage regulation:

$$\begin{aligned} \% \text{ voltage regulation} &= \frac{\text{No-load voltage} - \text{Full load voltage}}{\text{Full load voltage}} \times 100 \\ &= \frac{E_b - V}{V} \times 100 \end{aligned}$$

The voltage regulation is effected by three factors

- (i)  $I_a R_a$  drop (in armature winding)
- (ii)  $I_a X_L$  drop (in armature winding)
- (iii) voltage change due to armature reaction.

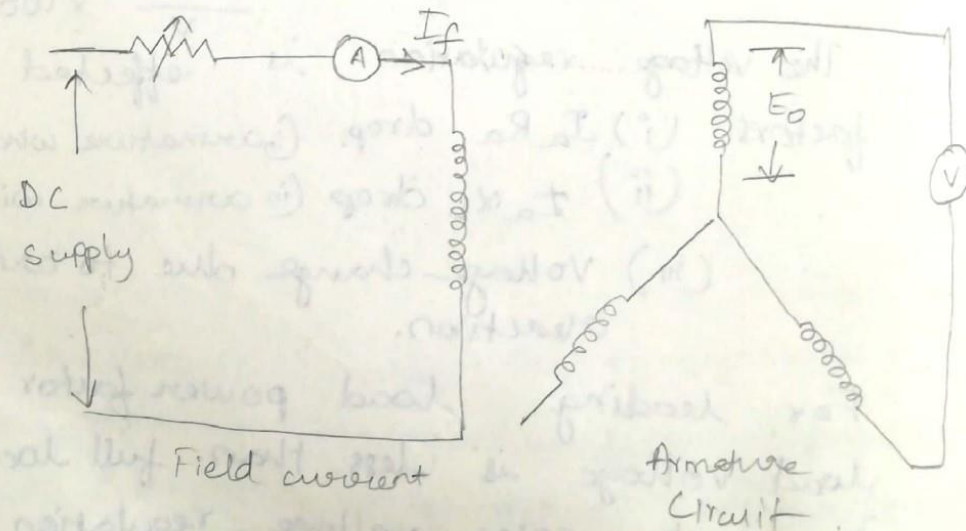
For leading load power factor the no load voltage is less than full load voltage. In such cases voltage regulation is -ve. For determining voltage regulation, there are three methods.



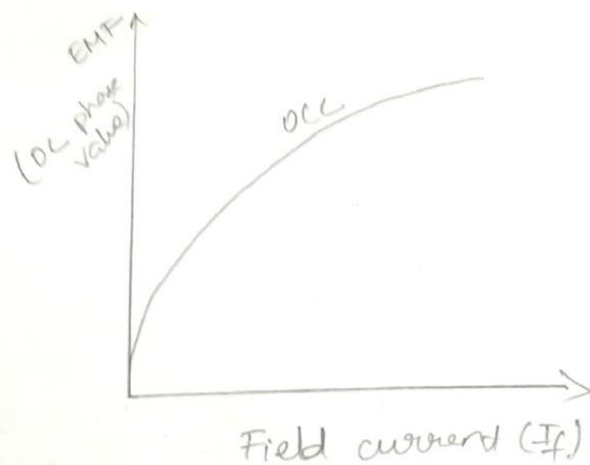
- ① Synchronous Impedance/EMF method
- ② Ampere turn/ MMF method.
- ③ zero power factor / Potier method

The data required are armature resistance, Open circuit characteristics (OCC) and short circuit characteristics (SCC). To find armature resistance,  $R_a$  we use direct current and voltmeter ammeter method.

To find OCC, OCC is obtained we draw a curve between, <sup>armature</sup> terminal voltage and field current. The armature terminal voltage is obtained by open circuiting and field current is taken when the alternator is running at rated speed.

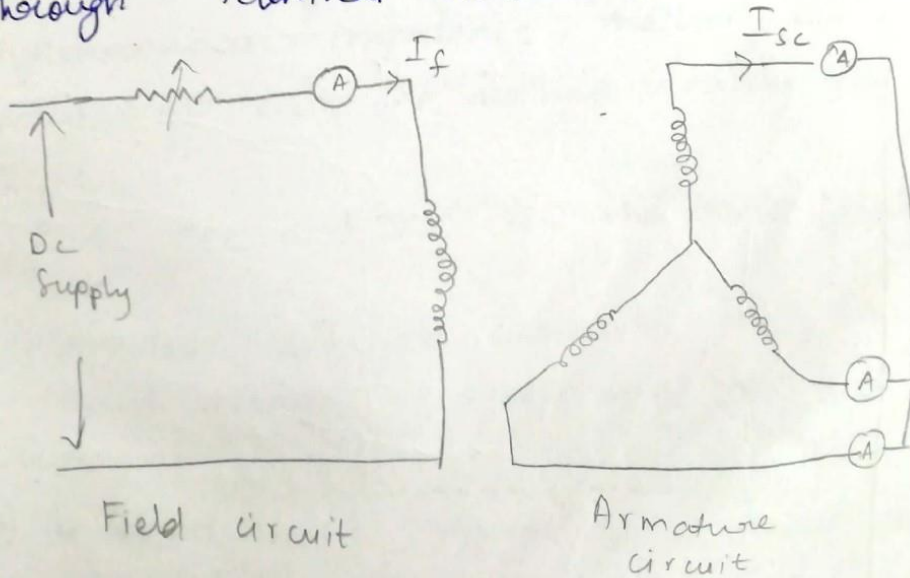


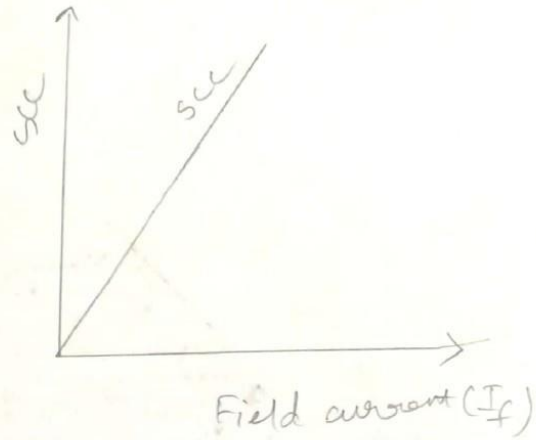




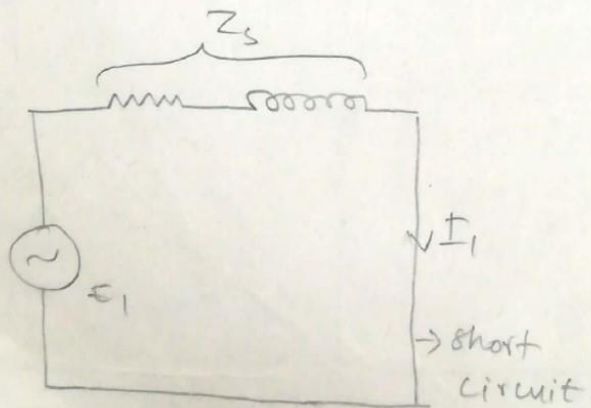
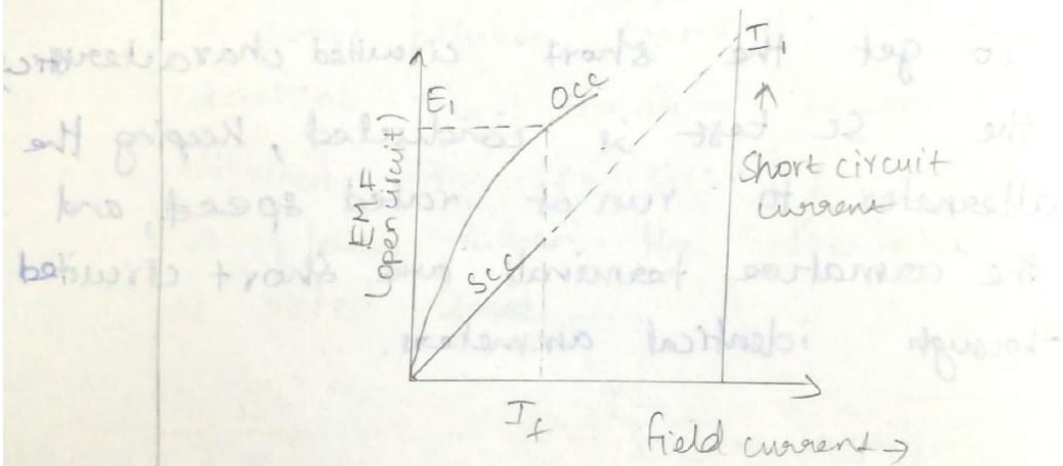
Short Circuit Characteristic: (SCC)

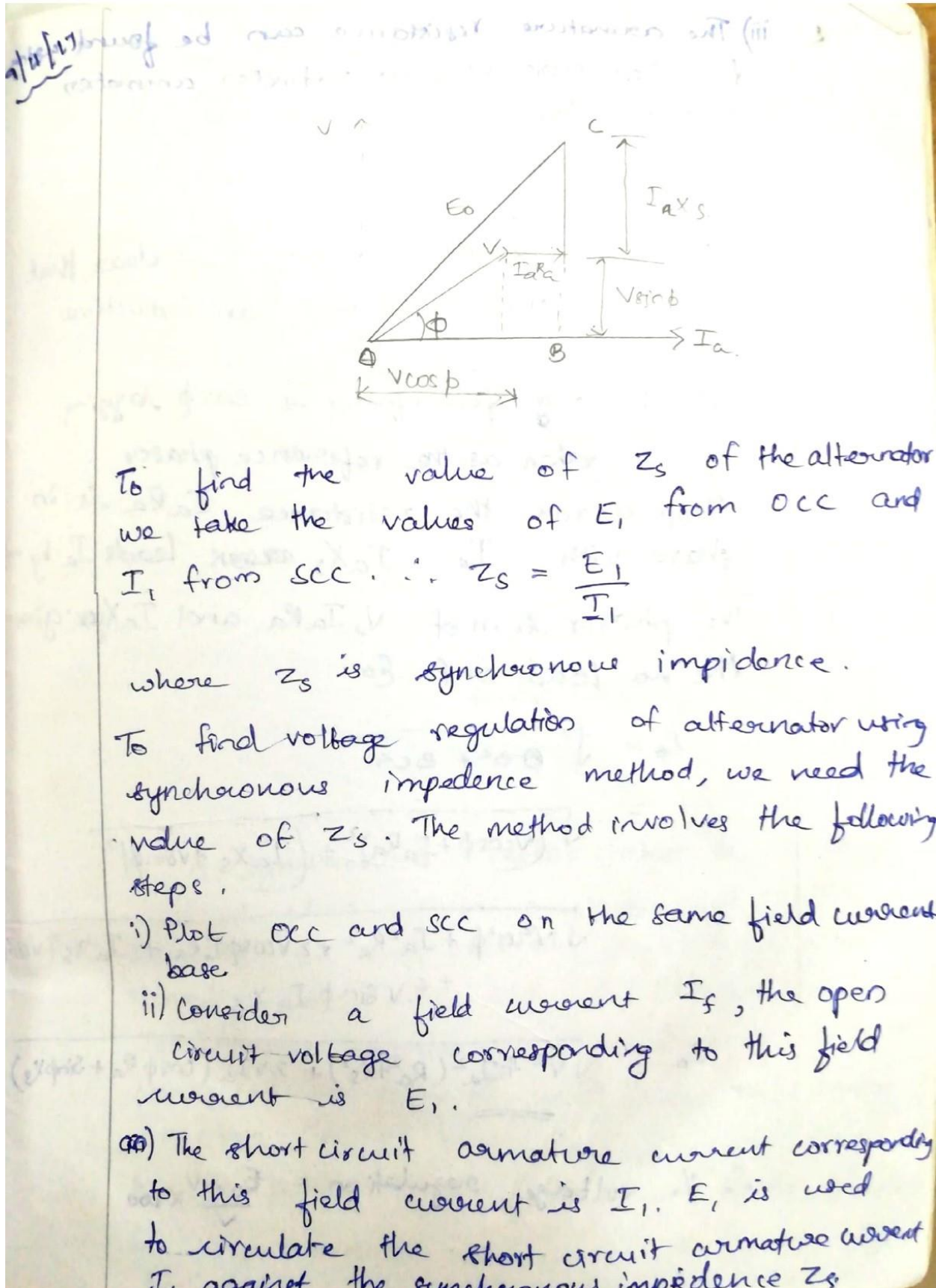
To get the short circuited characteristics, the SC test is conducted, keeping the alternator to run at rated speed, and the armature terminals are short circuited through identical ammeters.





Synchronous Impedance Method:







iii) The armature resistance can be found using the direct current and voltmeter, ammeter method. Synchronous reactance.

$$\therefore X_s = \sqrt{Z_s^2 - R_a^2}$$

iv) From the phasor diagram, it is clear that the load considered was an inductive load.

$\therefore$  The load power factor is  $\cos \phi$  lagging.

$I_a$  is taken as the reference phasor.

Drop across the resistance  $I_a R_a$  is in phase with  $I_a$ .  $I_a X_s$  lags  $I_a$  by  $90^\circ$ .

The phasor sum of  $V$ ,  $I_a R_a$  and  $I_a X_s$  gives the no load emf  $E_0$ .

$$E_0 = \sqrt{A^2 + B^2}$$

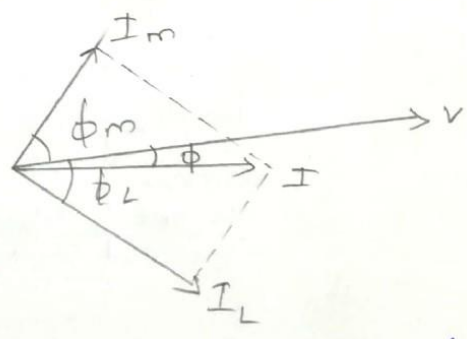
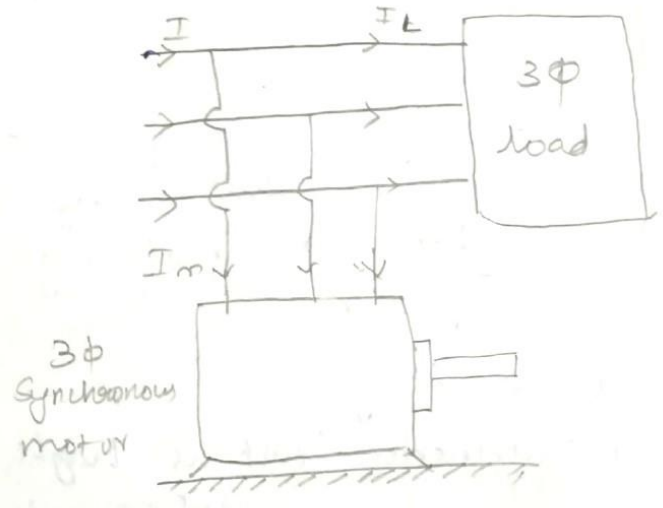
$$= \sqrt{(V \cos \phi + I_a R_a)^2 + (I_a X_s + V \sin \phi)^2}$$

$$= \sqrt{V^2 \cos^2 \phi + I_a^2 R_a^2 + 2V \cos \phi I_a R_a + I_a^2 X_s^2 + V^2 \sin^2 \phi + 2V \sin \phi I_a X_s}$$

$$E_0 = \sqrt{V^2 + I_a^2 (R_a^2 + X_s^2) + 2V I_a (\cos \phi R_a + \sin \phi X_s)}$$

$$\therefore \% \text{ voltage regulation} = \frac{E_0 - V}{V} \times 100$$

Synchronous Condenser: running with no load



A synchronous motor takes a leading current when over excited and behaves as a capacitor. An over excited synchronous motor running on no load is known as synchronous condenser. When these machines are connected in parallel with induction motor or other devices, that operates at low lagging power factor, leading KVAR, supplied by synchronous condenser partly neutralizes the lagging reactive KVAR of the load.

Thus the power factor of the system can be improved.

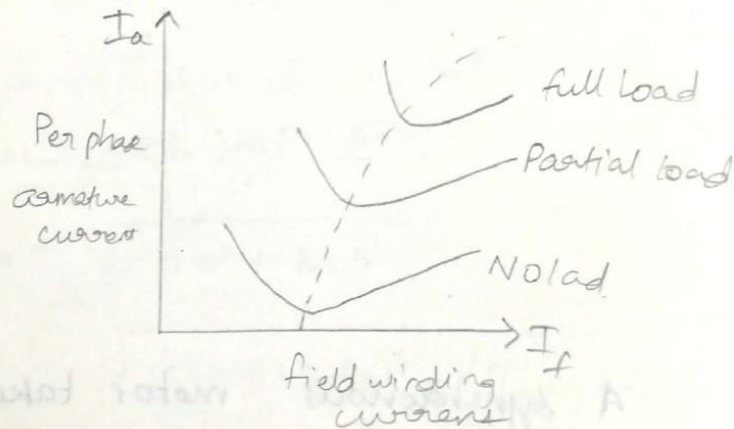
### Advantages:

By varying the field excitation, magnitude of current drawn by the motor can be changed which help in achieving stepless control of power factor.

### Disadvantages:

Maintenance cost is high.

### V-curves



The graph b/w armature current  $I_a$ , field current  $I_f$  of a synchronous motor for constant load is called v curve

When the level of excitation of synchronous motor is changed from under excitation to over excitation, for constant load, following are observed.

(i) When motor is under excited ( $E_b < V$ ),



the power factor is lagging and the motor behaves like inductive load.

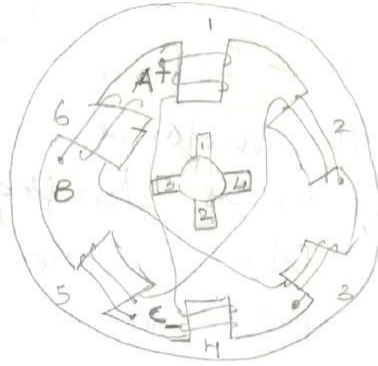
(ii) When the motor is normally excited. ( $E_b = V$ ). Power factor is unity. Armature current is minimum and in phase with terminal voltage.

(iii) When the motor is over excited. ( $E_b > V$ ), power factor is leading and the motor behaves like a capacitive load which improves power factor 3  $\phi$  supply.

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 Module - VI

### Variable Reluctance Stepper Motor :



It operates on the principle that when a piece of ferromagnetic material free to rotate is placed in a magnetic field Torque act on the material to bring it to the position of minimum reluctance to path of magnetic flux.

#### Construction:

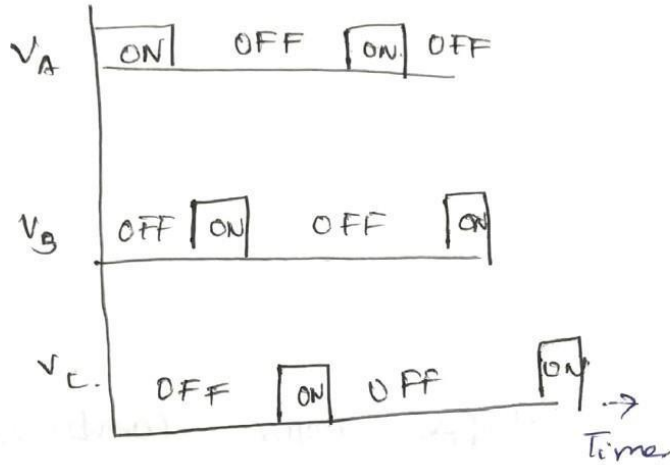
Stator is same as that of permanent magnet stepped motor. Stator phase windings are mount on each stator tooth. The rotor is made of soft steel with teeth and slot. In the figure, rotor is having fewer teeth than that of stator. This ensures that, one set of stator and rotor teeth will align at





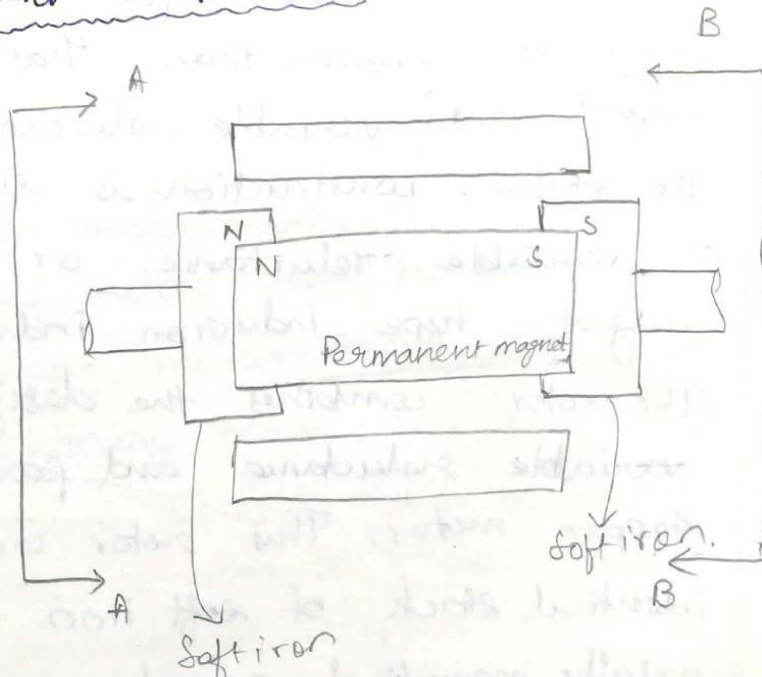
Cycle	Phase			Position
	0w	0Fr	OFF	0°
	ON	OFF	OFF	90°
H	OFF	ON	OFF	300°
	OFF	OFF	ON	330°

Voltage waveform



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Hybrid Stepper Motor.





of permanent magnet and variable reluctance stepper motor. The torque developed by this motor is greater than that of permanent magnet and variable reluctance type. The stator construction is similar to that

variable magnet type

induction design rotor permanent magnet two

identical stack of soft iron axially magnetized

well as magnet.

teeth are

on soft iron one end

north

teeth and

south

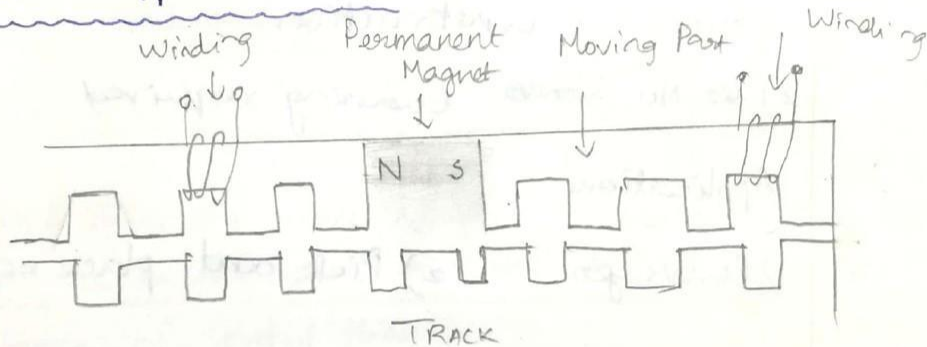


similar to permanent magnet or variable reluctance stepper motor. The four windings are energized in proper sequence and the motor rotates in step.

$$\text{The step angle } \alpha = \frac{90^\circ}{N_r}$$

where  $N_r$  is no. of rotor teeth.

### Linear Stepper motor:



Linear motor works on same principle as of rotating motors. Here instead of rotation it is linear motion. This motor consist of a stationary track and a moving assembly. The moving assembly has a no. of teeth that are similar to those found on motor in a conventional stepper motor. It has two set of field windings and a permanent magnets. The stationary track can be considered as stator of conventional stepper motor. When the field windings are energized, one set of teeth is aligned with teeth of stationary track.

The magnetic flux from the electromagnet aids the flux lines of one of the permanent magnets and cancel the flux lines of other permanent magnets. When the current flow to the coil is stopped, the moving assembly will align itself to the appropriate tooth set.

Advantage:

- 1) Simple in construction
- 2) No servos tuning required

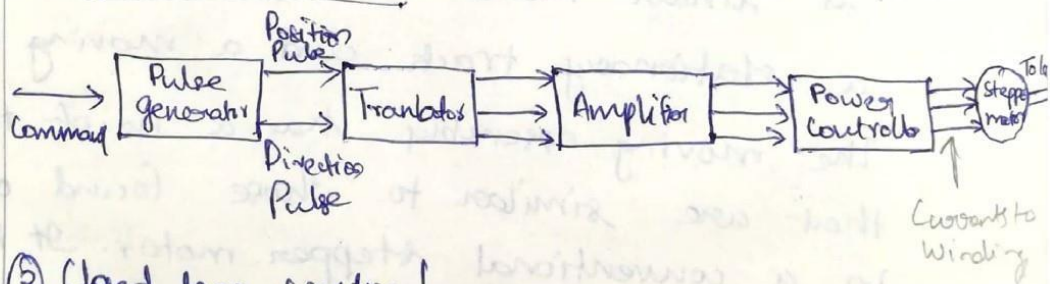
Applications:

- 1) Conveyers
- 2) Pick and place equipment.

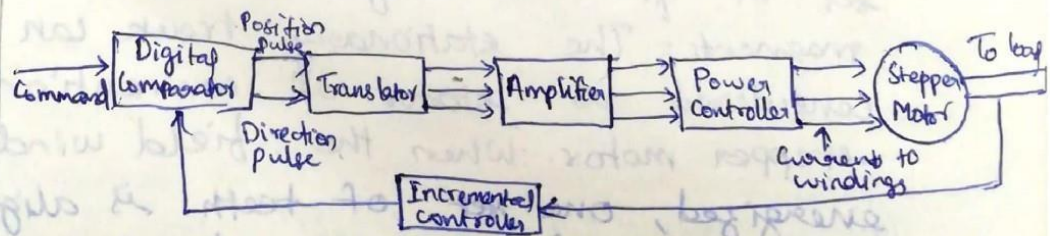
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Control of Stepper Motor

① Open loop control



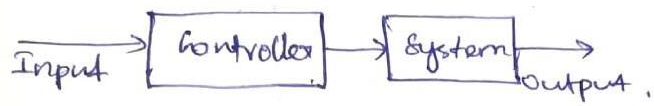
② Closed loop control





Control System

Open loop system



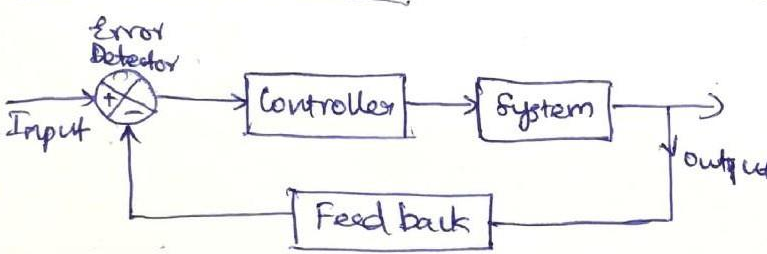
Merits (Open)

- It is simple
- Highly economical
- More stable than closed loop systems
- Maintenance are much less

Demerits

- They are inaccurate
- Unreliable; variation due to external disturbances are not corrected autom.

Closed loop system



Merits Closed

- They are less effect by non linearity & dist
- High accuracy
- Variation due to ext. dist. can be automatically correct.

Demerits: They are more complex compared to open loop system. The overall gain decreases due to presence of feedback.

Importance of Control theory

- Stability
- Feedback

• Transfer function =  $\frac{\text{Laplace transform of output}}{\text{Laplace transform of input}}$

$$G(s) = \frac{C(s)}{R(s)}$$

$G(s)$  = Gain of system       $C(s)$  = Output     $R(s)$  = Input

Order of a system

Suppose  $G(s) = \frac{20}{s^2 + 2s}$ . The order of system is maximum power of  $s$  in denominator polynomial i.e. it is 2.

Type of a system

Suppose  $G(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)\dots}{s^N(s+p_1)(s+p_2)(s+p_3)\dots}$



Here  $P_1, P_2, P_3$  are poles of transfer function  
 $Z_1, Z_2, Z_3$  are zeroes of transfer function  
 $N$  gives the type no. of the system

$$G(s) = \frac{K \omega}{s(s^2 + 2s + 2)} \quad \text{As } N=1, \text{ type} = 1$$

$K$  is a constant.

Zeroes  $\rightarrow (0)$

Poles  $(x)$  } in graph

